Real-Counter Automata and Their Decision Problems*
(Extended Abstract)

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Abstract. We introduce real-counter automata, which are two-way finite automata augmented with counters that take real values. In contrast to traditional word automata that accept sequences of symbols, real-counter automata accept real words that are bounded and closed real intervals delimited by a finite number of markers. We study the membership and emptiness problems for one-way/two-way real-counter automata as well as those automata further augmented with other unbounded storage devices such as integer-counters and pushdown stacks.

1 Introduction

An automaton is a finite-state language acceptor, that is possibly augmented with other unbounded storage devices like counters, stacks, and queues. Decision problems like membership and emptiness have been extensively studied in automata theory in the past 50 years. The membership problem is to decide whether a given word is accepted by an automaton, while the emptiness problem is to decide whether the language accepted by an automaton is empty. Studies on the decision problems have been one of the focuses in automata theory and have already benefited almost every area in computer science, including model-checking \cite{7,26} that seeks (semi-) automatic procedures to check whether a system design satisfies its requirements. Algorithmic solutions to decision problems like emptiness for various classes of automata (e.g., finite automata, Buchi automata, tree automata, pushdown automata, etc.) have become part of the theoretical foundation of model-checking finite-state/infinite-state systems. For instance, it is known that various model-checking problems such as LTL model-checking over finite-state transition systems and reachability for some infinite-state transition systems can be reduced to various emptiness problems (e.g., \cite{26,10}). Still, practitioners in formal specification/verification keep challenging automata theorists with new models emerging from verification applications. Some of the models, however, have not been

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well-studied in traditional automata theory. A typical example concerns the theory and fundamental verification techniques for analyzing hybrid transition systems containing both real variables (e.g., to model time, water level, etc.) and other unbounded discrete data structures (e.g., to model the number of times a request is sent, the call stack of a recursive process, etc.). To this end, in this paper, we study real-counter automata which contain counters that take real values.

In contrast to a traditional word automaton, a two-way real-counter automaton works on a real word provided on the input tape. A real word is a bounded and closed real interval (like $[0,10]$), in which the two end points and a finite number of other given (intermediate) points are called markers. Each marker as well as each segment between two consecutive markers is labeled with a color drawn from a finite color set. The automaton scans through the input real word in a two-way fashion, and can distinguish whether the current read head is over a marker or within a segment. The automaton can also recognize the color of the corresponding marker/segment. During the scan, each real-counter stays unchanged or is incremented/decremented (according to the instruction that is being executed) for an amount equal to the “distance” the head moves. The automaton can also test a real-counter against 0.

In this paper, we focus on membership and emptiness problems for two-way real-counter automata ($\mathcal{R}$-2NCMs). In general, these problems are undecidable, since $\mathcal{R}$-2NCMs automata have Turing computing power. Therefore, we study some restrictions that can be applied to the model to obtain decidable membership/emptiness problems. For instance, we show decidability for $\mathcal{R}$-2NFAs (i.e., $\mathcal{R}$-2NCMs that do not have real-counters). Another restriction is reversal-boundedness: a real-counter is reversal-bounded (r.b. for short) if the counter changes modes between nondecreasing and non-increasing for at most a fixed number of times during any computation. We use r.b. $\mathcal{R}$-2NCMs to denote $\mathcal{R}$-2NCMs where each real-counter is reversal-bounded. We show that the membership problem for r.b. $\mathcal{R}$-2NCMs is decidable, while the emptiness problem is undecidable. The latter undecidability remains even when the input real-words have only $k$ markers, for a fixed $k$. We also study the decision problems for various versions of one-way real-counter automata. In particular, we study one-way/two-way real-counter automata that are further augmented with other unbounded discrete storages like integer-counters and/or a pushdown stack. Some of our decidability results make use of mixed linear constraints over both integer variables and real variables and the concept of mixed semilinearity over a language of real words. The concept generalizes the traditional notion of semilinearity [23] over a language of words. This makes it convenient for us to study various classes of one-way/two-way real-counter automata that have a mixed accepting condition which is a Boolean combination of mixed linear constraints over real-counters and integer-counters.

The rest of the paper is organized as follows. Section 2 defines basic notations and introduces some known results on integer-counter automata. Section 3 studies the membership and emptiness problems for two-way real-counter automata. Section 4 presents decidability results on one-way real-counter automata, also further augmented with integer-counters and a pushdown stack. Section 5 is a brief conclusion, also outlining possible applications of the real counter model to the verification of classes of hybrid systems.