$\in_T$-Integration of Logics

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Abstract. $\in_T$-logic was first designed by Werner Sträter as a first-order propositional logic with quantification, reference, and predicates for *true* and *false*. It is motivated by reconstruction of natural language semantics and allows, as a logic with self-reference and impredicativity, among others the treatment of the liar paradox despite the totality of its truth predicates. Its intensional models form a theory of propositions for which a correct and complete calculus is given.
$\in_T$-logic was picked up by Philip Zeitz to study the extension of abstract logics by the concepts of truth, reference and classical negation, thereby rebuilding the meta-level of judgements in a formal level of propositional logic. His parameterized $\in_T$-logic allows formulas from a parameter logic to become the constants in his $\in_T$-logic. Parameter-passing of logics with correct and complete calculus also admits, under certain conditions, the entailment of a calculus which is correct and complete for the extended logic.
Since in parameterized $\in_T$-logic Tarski Biconditionals not only apply for the truth of $\in_T$-logic sentences, but also for the meta-level truth of the parameter logic it is natural to view $\in_T$-logic as a theory of judgements whose propositions are expressed in the parameter logic.
We add a new interpretation to $\in_T$-logic as a theory of truth and judgements, and introduce $\in_T$-logic as a means for the integration of logics. Based on a particular choice of uniform view and treatment of logics we define $\in_T$-logics and $\in_T$-extensions as the foundation for $\in_T$-integration of logics and models.
Studies in $\in_T$-logic, which have started to deal with the difficulties of truth in natural language semantics, have evolved into a concept of logic integration where application oriented logics can be plugged in as parameters. This paper very much relies on the work of Philip Zeitz, but opens it for the new perspective of integration.

1 Introduction

There are various approaches to integration in the design and description of systems. Here integration of logics has the goal to define a logic formalism which admits compound propositions whose components come from different logics. For this task of integration $\in_T$-logic is used as a basis.

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1.1 Studies in $\in_T$-Logic

$\in_T$-logic, a theory of truth and propositions, was first defined by Werner Sträter in the context of reconstructing natural language semantics by means of self-referential structures (see [15, 14]). Based on propositional variables and constants formulas of $\in_T$-logic are built from classical propositional connectives together with quantification over propositional variables, propositional equality and predicates for truth and falsity. The resulting logic is intensional in that it has not just truth values but rather sets of propositions as its models. It has been proven free from antinomies despite its total truth-predicates and its ability to model self-referential sentences and impredicative quantification. This is shown in a model existence theorem. Also a correct and complete calculus of $\in_T$-logic is given by Sträter. The concept of $\in_T$-integration defined below is a reinterpretation and particular use of $\in_T$-logic.

Logic reconstruction of natural language semantics shows the need to simultaneously handle various kinds of logics as well as their extension by concepts of truth and reference. This need was taken as a motivation for a generalization of $\in_T$-logic, which admits propositional constants to be formulas from some other logic. The study of this generalization resulted in the definition of the so-called $\in_T$-extension by Philip Zeitz (in [26]). To deal with concrete extensions of arbitrarily given logics Zeitz studies different forms of abstract logics and introduces a particular form, in which the semantics is given by a system of sets, called basis. For extensions of such logics in abstract form Zeitz studies the existence of models and the conditions which allow to extend a correct and complete calculus of the parameter logic so that the resulting calculus is correct and complete for the extended logic. He also discusses basics of a model theory and the existence of intensional propositional interpretations.

$\in_T$-logic has proven to be a suitable concept for truth and reference. It avoids antinomies which necessarily appear with logics having total truth predicates and at the same time allow for representations of decidable relations and computable functions (Tarski, see [24]). In this respect $\in_T$-logic is not a rich language. The concept of $\in_T$-extension however circumvents limitations in expressive power by some kind of stratification in which language features that are rich enough for representations are being separated from their conceptual truth predicates.

1.2 Concepts of Integration

The task of specification has more and more become a task of dealing with the complexity of the system to be specified. Early approaches to cope with complex system structures were based on the concept of modularization, first studied in the context of programming (Parnas, see [18]) and later in the context of abstract data type specification (see Goguen, Tardo [12] and Ehrig, Mahr [4, 5]). Integration was here understood as composition of specifications based on some underlying logic. The need of a variety of logics for specification has motivated the study of abstract concepts of system description in appropriate formal or conceptual frameworks. Initially these frameworks were specification languages based