Loose Semantics of Petri Nets*

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Abstract. In this paper, we propose a new,loose semantics for place/transition nets based on transition systems and generalizing the reachability graph semantics. The loose semantics of a place/transition net reflects all its possible refinements and is given as a category of transition systems with alternative sequences of events over the net. The main result states that each plain morphism between two place/transitions nets induces a free construction between the corresponding semantic categories.

1 Introduction

Petri nets are one of the most thoroughly investigated approaches with a multitude of extensions and variants. They are one of the most prominent specification techniques for modeling concurrency and have a wide range of application areas in practice. In this paper, we introduce a new semantics for Petri nets which is based on transition systems. The semantics of a net is given by a class of models corresponding to all possible refinements of a net with respect to transition refinement. In this sense, it is a loose semantics as known and well accepted in the area of data type specification (see, e.g. [24]).

The semantics we define here is developed in view of system specification. It is suitable for relating different stages of refinement. This is obviously important for the vertical structuring, but as well for horizontal structuring with abstraction mechanisms like parameterization and modularization.

The reachability graph is a standard model of a place/transition net describing all possible sequences of firings of transitions starting from an initial marking. Our new semantics generalizes this net semantics in such a way that a firing of a transition can be refined by sequences of events. Moreover, we allow alternative possibilities for each such refinement. Typical examples of alternative sequences are the interleavings of independent events. Altogether, the loose semantics of a place/transition net consists of the class of transition systems with alternative sequences of events including the reachability graph. This class forms a category

* Research partially supported by the EC Research Training Network SegraVis.

H.-J. Kreowski et al. (Eds.): Formal Methods (Ehrig Festschrift), LNCS 3393, pp. 370–384, 2005.

in a natural way. As the main result of the paper, we show that each plain morphism between two place/transition nets induces a free construction between the corresponding semantic categories. This is the key result that allows one to consider Petri nets as building blocks of parameterization and modularization.

We continue this paper by introducing state transition systems that capture our idea of alternatives and refinement. In Section 3, we show that transitions systems over Petri nets can be considered as a loose semantics of the corresponding Petri net. Next we show that we obtain a free functorial construction of the place/transition net semantics, based on contravariant forgetful functor. Subsequently in Section 5 we treat the relation to other approaches in some detail and discuss at last the impact of a loose semantics for Petri nets in Section 6.

2 Transition Systems with Alternative Sequences

In this section, we recall the notion of state transition systems and add a new feature to them: a relation of alternative sequences of events. Transition systems with alternative sequences will be combined with place/transition nets in the next section.

A state transition system $STS = (S, E, TS, \widehat{s})$ is given by a set of states S, a set of events E, the set of transitions $TS \subseteq S \times E \times S$, and the initial state $\widehat{s} \in S$.

If one reads the events along the paths in state transition systems, one gets sequences of events. More formally, we write $s_0 \xrightarrow{w} s_n$ if there is some sequence of transitions $(s_{i-1}, e_i, s_i) \in TS$ for $i = 1, .., n$ and $w = e_1 e_2 ... e_n \in E^*$ or if $w = \lambda$ and $s_0 = s_n$.

Next we want to consider some of these sequences of events as alternatives to each other. To make this precise, let $AS \subseteq E^* \times E^*$ be some relation on E^* and AS^{Con} denote its congruence closure, i.e. the closure of AS that is reflexive, symmetric, transitive, and congruent with respect to concatenation. Moreover, let $E^\diamond$ denote the quotient factoring E^* through AS^{Con} and $[\_] : E^* \to E^\diamond$ the canonical function with $[\_](w) = [w]$ for all $w \in E^*$ where $[w] = \{w' | (w, w') \in AS^{Con}\}$ is the congruence class of $w \in E^*$.

Definition 1 (Transition Systems with Alternative Sequences). *A transition system with alternative sequences $TSA = (S, E, TS, \widehat{s}, AS)$ is given by a state transition system $(S, E, TS, \widehat{s})$ and the relation of alternative sequences $AS \subseteq E^* \times E^*$ subject to the following consistency condition:*

$$\forall w' \in [w] : s \xrightarrow{w} s' \iff s \xrightarrow{w'} s'$$

The consistency condition ensures that alternatives are alternatives at all states. So, they are global alternatives in the following sense: Whenever there is a state where the sequence w occurs the alternative sequence $w' \in [w]$ has to occur as well.

Next we examine morphisms between transition systems with alternative sequences. We allow mapping one event $e_1 \in E_1$ to a congruence class of sequences of events by a morphism $f_E : E_1 \to E_2^\diamond$ with $f_E(e_1) = [w]$. This