Dynamic Archive Evolution Strategy for Multiobjective Optimization

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Abstract. This paper proposes a new multiobjective evolutionary approach—the dynamic archive evolution strategy (DAES) to investigate the adaptive balance between proximity and diversity. In DAES, a novel dynamic external archive is proposed to store elitist individuals as well as relatively better individuals through archive increase scheme and archive decrease scheme. Additionally, a combinatorial operator that inherits merits from Gaussian mutation of proximity exploration and Cauchy mutation of diversity preservation is elaborately devised. Meanwhile, a complete nondominance selection ensures maximal pressure of proximity exploitation while a corresponding fitness assignment ensures the similar pressure of diversity preservation. By graphical presentation and performance metrics on three prominent benchmark functions, DAES is found to outperform three state-of-the-art multiobjective evolutionary algorithms to some extent in terms of finding a near-optimal, well-extended and uniformly diversified Pareto optimal front.

1 Introduction

A formal notion of multiobjective optimization is given by Fonseca and Fleming in [1]. Without loss of generality, consider the following multiobjective optimization with n decision variables x and m (m>1) objectives y:

$$\begin{aligned} &\min \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = \{f_1(x_1,\cdots,x_n),\cdots,f_m(x_1,\cdots,x_n)\} \\ &s.t. \quad \boldsymbol{x} = (x_1,\cdots,x_n) \in \boldsymbol{X} \subset R^n \\ &\qquad\;\; \boldsymbol{y} = (y_1,\cdots,y_m) \in \boldsymbol{Y} \subset R^m \end{aligned} \tag{1}$$

where $\boldsymbol{x}$ is called decision vector, $\boldsymbol{X}$ decision space, $\boldsymbol{y}$ objective vector and $\boldsymbol{Y}$ objective space; $\boldsymbol{f}$ defines the mapping function. The scenario considered in this paper involves an arbitrary optimization with objectives, which are all to be minimized and all equally important. It means that the objectives cannot be combined into a single scalar objective to be optimized, so the sets of solutions exist such that each solution in this set is equally preferable. The following four concepts are of importance:

1. **Pareto dominance:** A solution $\boldsymbol{x}^0$ is said to *dominate* (*Pareto optimal*) another solution $\boldsymbol{x}^1$ (denoted $\boldsymbol{x}^0 \succ \boldsymbol{x}^1$) if and only if:
   $\forall i \in \{1,\cdots,m\} : f_i(\boldsymbol{x}^0) \le f_i(\boldsymbol{x}^1) \wedge (\exists k \in \{1,\cdots,m\} : f_k(\boldsymbol{x}^0) < f_k(\boldsymbol{x}^1))$.

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2. **Pareto optimal:** A solution $\boldsymbol{x}^0$ is said to be *nondominated* (*Pareto optimal*) if and only if: $\neg\exists \boldsymbol{x}^1 \in \boldsymbol{X} : \boldsymbol{x}^1 \succ \boldsymbol{x}^0$.
3. **Pareto optimal set:** The set P_S of all Pareto optimal solutions: $P_S = \{\boldsymbol{x}^0 \mid \neg\exists \boldsymbol{x}^1 \in \boldsymbol{X} : \boldsymbol{x}^1 \succ \boldsymbol{x}^0\}$.
4. **Pareto optimal front:** The set P_F of all objective function values corresponding to the solutions in P_S: $P_F = \{\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})) \mid \boldsymbol{x} \in P_S\}$.

The optimal result for such multiobjective optimization is no other than the Pareto optimal set P_S. However, the size of this set may be infinite, and it is impossible to find this set by using a finite number of solutions. In this case, a representative subset of P_S is desired. Generally, the characteristic of multiobjective evolutionary algorithms (MOEAs) is to search the decision space by maintaining a finite population of individuals (corresponding to the points in the decision space), which work according to the procedures that resemble the principles of natural selection and evolution. Because we only consider the subset of all the final nondominated individuals resulted from a MOEA, we call such subset an *approximation set* and denote it by *S*, and we call the corresponding objective set a *resulting final Pareto optimal front* and denote it by PF_{final}. Ideally; we are interested in finding an *S* of finite size, which contains a selection of individuals from such that the individuals in PF_{final} are diversified as possible. Unfortunately, we usually have no access to P_F on beforehand. We have to get close to P_F but in such a way that PF_{final} we found is as diversified as possible without compromising as much as possible the proximity of PF_{final} with respect to P_F. Thus, the concept of *proximity* and *diversity* should be outlined. Regarding this diversity, it is of importance to note that it depends on the mapping function whether a good diversity of the individuals in the decision space is also a good diversity of the individuals in the objective space correspondingly. However, it is common practice to search for a good diversity of the individuals in the objective space because decision makers will ultimately have to pick a single individual as final solution according to its objective vector values. Therefore, it is often best to present a wide variety of tradeoff individuals for the specified goals in constructing MOEAs.

During the past decade, various MOEAs have been proposed and applied [1]. A representative collection of these influential algorithms includes the Nondominated Sorting Genetic Algorithm (NSGA) and NSGA2 by Srinivas and Deb et al [2] [3], the Strength Pareto Evolutionary Algorithm (SPEA) and SPEA2 by Zitzler et al [4] [5], the Pareto Archived Evolution Strategy (PAES) and the memetic PAES (M-PAES)by Knowles and Corne [6] [7] etc. Although these MOEAs differ from each other, they share the common purpose — searching for a near-optimal, well-extended and uniformly diversified PF_{final} for a given multiobjective optimization. However, this ultimate goal is far from being accomplished by the existing MOEAs as documented in the literature, e.g., [1],[5]. In one respect, most of multiobjective optimizations are very complicated and have their own inherent characters and variabilities, so computational resources are required to be homogenously distributed in a high-dimensional decision space. On the other hand, those fitter individuals generally have strong tendencies to restrict searching efforts within local areas because of the genetic drift phenomenon [8], which results in the loss of diversity. This highlighted the hot issue how to improve the algorithm's robustness and how to balance proximity and diversity during searching process. However, the latter issue we considered in this paper has not