Searching for Robust Pareto-Optimal Solutions in Multi-objective Optimization

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Abstract. In optimization studies including multi-objective optimization, the main focus is usually placed in finding the global optimum or global Pareto-optimal frontier, representing the best possible objective values. However, in practice, users may not always be interested in finding the global best solutions, particularly if these solutions are quite sensitive to the variable perturbations which cannot be avoided in practice. In such cases, practitioners are interested in finding the so-called *robust* solutions which are less sensitive to small changes in variables. Although robust optimization has been dealt in detail in single-objective optimization studies, in this paper, we present two different robust multi-objective optimization procedures, where the emphasis is to find the robust optimal frontier, instead of the global Pareto-optimal front. The first procedure is a straightforward extension of a technique used for single-objective robust optimization and the second procedure is a more practical approach enabling a user to control the extent of robustness desired in a problem. To demonstrate the subtle differences between global and robust multi-objective optimization and the differences between the two robust optimization procedures, we define four test problems and show simulation results using NSGA-II. The results are useful and should encourage further studies considering robustness in multi-objective optimization.

1 Introduction

For the past decade or more, the primary focus of the research and application in the area of evolutionary multi-criterion optimization (EMO) has been placed in finding the globally best Pareto-optimal solutions. Such solutions are non-dominated to each other and there exists no other solution in the entire search space which dominates any of these solutions. From a theoretical point of view, such solutions are of utmost importance in a multi-objective optimization problem. However, in practice, often a solution cannot be implemented with arbitrary precision for various reasons and the implemented solution may be slightly different from the desired solution. If a global optimal solution is *sensitive* to variable perturbation in its vicinity, the implemented solution may correspond to different objective values than that of the theoretical optimal solution. Thus,

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from a practical standpoint, such solutions are of not much importance and the emphasis must be made in finding *robust* solutions, which are less sensitive to variable perturbations in their vicinity.

In single-objective optimization, a number of studies have been devoted for finding such robust solutions. Branke [1] suggested a number of heuristics for searching robust solutions. In another study, Branke [2] suggested a number of methods for alternate fitness estimation. Later, Branke [1] also pointed out key differences between searching optimal solutions in a noisy environment and searching for robust solutions. Jin and Sendhof [3] posed the issue of finding robust solutions in single-objective optimization problem as a multi-objective optimization problem with the objectives being maximizing robustness and performance. Tsutsui and Ghosh [4] presented a mathematical model for obtaining robust solutions using schema theorem for single-objective genetic algorithms. Parmee [5] suggested a hierarchical strategy of searching several high performance regions in a fitness landscape simultaneously. Teich [6] extended Pareto-dominance for handling uncertain objectives and Hughes [7] computed the error estimate for using deterministic Pareto-dominance in noisy functions. However, to our knowledge, there does not exist a systematic study introducing robustness in evolutionary multi-objective optimization.

In this paper, we make an effort to extend an existing approach for finding robust solutions in single-objective optimization for multi-objective optimization. Essentially, in this approach, instead of optimizing the original objective functions, we optimize the mean effective objective values computed at a point by averaging the function values of a few solutions in its vicinity. The solutions which are less sensitive to local perturbations will fair well in terms of the mean effective objective values and the resulting Pareto-optimal front will be the robust frontier. To illustrate the working of this approach, we first suggest four different controllable test problems and then employ NSGA-II. We also present a new definition of robustness in which original objectives are optimized but a constraint limiting the change in function values due to local perturbations is added. The latter approach is more pragmatic and a user has a control on the desired level of robustness on the obtained solutions. The differences between these two robust procedures and fundamental differences between global and robust optimization in the context of multi-objective optimization are clearly demonstrated.

Rest of the paper is designed as followed. Section 2 introduces the concept of robustness in multi-objective optimization and stresses its importance. Sections 3 and 4 discuss the two robust optimization schemes and results obtained using NSGA-II. Finally, a conclusion of this study is presented in Section 5.

2 Robustness in Optimization

We consider a multi-objective optimization problem of the following type:

$$\left.\begin{array}{r}\text{Minimize } (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})),\\ \text{subject to } \mathbf{x} \in \mathcal{S},\end{array}\right\} \tag{1}$$