# Multi-objective Go with the Winners Algorithm: A Preliminary Study

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**Abstract.** This paper introduces a new algorithm to deal with multi-objective combinatorial and continuous problems. The algorithm is an extension of a previous one designed to deal with single objective combinatorial problems. The original purpose of the single objective version was to study in a rigorous way the properties the search graph of a particular problem needs to hold so that a randomized local search heuristic can find the optimum with high probability. The extension of these results to better understand multi-objective combinatorial problems seems to be a promising line of research. The work presented here is a first small step in this direction. A detailed description of the multi-objective version is presented along with preliminary experimental results on a well known combinatorial problem. The results show that the algorithm has the desired characteristics.

## 1 Introduction

Real world optimization problems are usually multi-objective (MO) in nature. The lack of exact methodologies to tackle these problems makes them attractive theoretically, and practically.

In the last few years many population-based strategies have been proposed to deal with these problems. These strategies seem especially well suited for MO problems where a set of solutions, instead of a single solution, is needed. Among them Genetic Algorithms [1, 9] have been the most widely used. Other approaches were proposed as an extension of their mono-objective counter part, some of these are: MO Simulated Annealing [14, 15], MO Ant Colony Optimization [16, 17], MO Particle Swarm Optimization [18], MO Evolutionary Strategy [21], MO Tabu Search [19, 20], among others.

The mainstream of research with these methods has been directed to experimental rather than theoretical issues. The reasons for this are simple: first, the analysis of these algorithms are extremely complex and the insight gained with the results are usually poor; second, the urgent need for methodologies to solve real problems is high.

C. A. Coello Coello et al. (Eds.): EMO 2005, LNCS 3410, pp. 206–220, 2005.

On the other hand the research community in analysis of algorithms has focused in trying to find a model for the random local search heuristics. This with the objective of giving a rigorous explanation of these algorithms behavior. One such attempt has been the model of heuristics where good solutions are preferred over bad solutions in what was called the "Go with the Winner" (GWW) scheme.

The GWW algorithm have been introduced by Aldous and Vazirani [2] as an attempt to give a rigorous explanation of the behavior of some algorithms based on a non crossover "survival of the fittest" paradigm. This algorithm resembles the well known simulated annealing strategy and is restricted to work with search graph with a tree structure [2]. Dimitriou and Impagliazzo [5, 6] generalize the algorithm to deal with general search graphs. In this paper an extension of the generalized version of GWW algorithm to deal with MO optimization problems is presented.

In order to extend all theoretical results of the single objective case to the MO case, a great deal of work will be required. Our first step is to propose an extension of the algorithm and to study its performance on a $NP$-*hard* scheduling problem.

Scheduling appears to be one of the most challenging combinatorial MO problems. In a real scheduling problem we are interested not only in minimizing the latest completion time (makespan) but also in minimizing the total time all jobs exceed their respective due dates. These objectives are usually in conflict with each other.

The remainder of the paper is organized as follows. Section 2 presents the GWW algorithm and extends it to deal with MO problems. Section 3 states the problem to be used as a study case. Section 4 shows the experimental setup and results. Finally, section 5 presents the summary of this work.

## 2 The Go with the Winners Algorithm

Originally, the GWW algorithm was designed for search graphs with a tree structure and the main result is as follows [2]. If we want to increase the success probability in a randomized optimization heuristic from $p$ (generally small) to 0.99, one way is to re-run the heuristic $O(1/p)$ times and select the best answer. In case of the GWW algorithm this number can be reduced to $log(1/p)$ runs of the algorithm by introducing some interactions between the runs. Dimitriou and Impagliazzo [5] introduced a variant of this algorithm that can be used for search graphs that are not trees. They show that for any one of a large class of distributions on search graphs their version of the GWW finds, in polynomial time, the optimum with high probability, for almost all search graphs drawn from the distribution. The most important result is that they also found a sufficient condition on the search graph structure (a combinatorial property) for the algorithm to reach the optimum with high probability. This property is known as the "local expansion property." Carson [3] uses the GWW algorithm to analyze, experimentally as well as analytically the search graph for the bisection problem. The analysis performed can give information regarding which instances will be