Exploring the Performance of Stochastic Multiobjective Optimisers with the Second-Order Attainment Function

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Abstract. The attainment function has been proposed as a measure of the statistical performance of stochastic multiobjective optimisers which encompasses both the quality of individual non-dominated solutions in objective space and their spread along the trade-off surface. It has also been related to results from random closed-set theory, and cast as a mean-like, first-order moment measure of the outcomes of multiobjective optimisers. In this work, the use of more informative, second-order moment measures for the evaluation and comparison of multiobjective optimiser performance is explored experimentally, with emphasis on the interpretability of the results.

1 Introduction

Stochastic multiobjective optimisers, such as evolutionary algorithms and other metaheuristics, produce Pareto-set approximations which consist of sets of non-dominated points in objective space. Given the stochastic nature of the optimisers, such non-dominated point sets may be seen as realisations of corresponding random non-dominated point sets, the stochastic behaviour of which is tied both to the problem and to the optimiser considered.

The performance of a multiobjective optimiser is intimately related to the quality of the Pareto-set approximations it produces. In the literature, several attempts have been made to quantify the quality of (deterministic) Pareto-set

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approximations through so-called unary quality indicators, which are functions which assign real values to each Pareto-set approximation. In the face of *random* Pareto-set approximations, unary quality indicators provide a convenient transformation from random sets to random variables. However, this approach suffers from inherent limitations of unary quality indicators. In fact, even a finite number of unary quality indicators which, in combination with each other, would completely describe a deterministic set of non-dominated points in objective space cannot exist in practice [1]. As a result, information is irremediably lost by finite-dimensional unary quality indicators even before any statistical analysis takes place.

To be able to retain all of the information available in the original non-dominated sets, a quality indicator must be infinite-dimensional, such as the binary field [2] derived from the set of goals attained by a Pareto-set approximation (attained set or attained region). Describing a Pareto-set approximation by a (real-valued) function defined over the whole of the objective space may not seem to be very sensible in the deterministic case. However, in the random case, it provides a useful link to existing random set theory, where distributions of random sets are studied directly (possibly, up to a complete distributional description) and not indirectly through distributions of summary measures (indicators) of the sets. The attainment function of a random Pareto-set approximation, for example, has been identified as the first-order moment measure of the binary random field derived from the corresponding random attained set [3], and, as such, is a concept perfectly integrated in random set theory.

By combining fundamentally different features of the quality of Pareto-set approximations, such as the quality of individual solutions and their spread along the trade-off surface, into a real-valued function of the goals, the attainment function can already effectively describe an important aspect of the distribution of random Pareto-set approximations, namely its location. To address the dependence structure of individual solutions within these approximation sets, two additional measures of performance are considered in this work, both of which are of second-order moment type: the second-order attainment function and its centred version, the covariance function.

In section 2, background is given on the attainment function. In section 3, the second-order attainment function and the covariance function are introduced. Empirical estimates of the two functions, obtained from experimental data, are presented graphically, and their interpretation is discussed. Section 4 is devoted to the comparison of optimiser performance using statistical hypothesis tests based on first-order or second-order attainment functions. To illustrate the application of the attainment function approach, experimental results obtained on two different optimisation problems are presented and discussed in section 5. The paper concludes with some remarks and directions for further work in section 6.

2 Background

The outcome of a multiobjective optimiser is considered to be the set of non-dominated objective vectors evaluated during one optimisation run. If the opti-