Comparing Classical Generating Methods with an Evolutionary Multi-objective Optimization Method

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Abstract. For the past decade, many evolutionary multi-objective optimization (EMO) methodologies have been developed and applied to find multiple Pareto-optimal solutions in a single simulation run. In this paper, we discuss three different classical generating methods, some of which were suggested even before the inception of EMO methodologies. These methods specialize in finding multiple Pareto-optimal solutions in a single simulation run. On visual comparisons of the efficient frontiers obtained for a number of two and three-objective test problems, these algorithms are evaluated with an EMO methodology. The results bring out interesting insights about the strengths and weaknesses of these approaches. Further investigations of such classical generating methodologies and their evaluation should enable researchers to design a hybrid multi-objective optimization algorithm which may be better than each individual method.

1 Introduction

Multi-objective optimization has been a rapidly growing area in modern optimization. There exist a plethora of methods and algorithms for solving multi-objective optimization problems. The methods can be divided in two categories: (i) classical methods which use direct or gradient-based methods following some mathematical principles and (ii) non-traditional methods which follow some natural or physical principles. Of them, the evolutionary multi-objective optimization (EMO) has been getting growing attention over the past decade. The classification is also appropriate from two other perspectives. The classical approaches usually use deterministic transition rules, whereas non-traditional approaches use stochastic rules. They are also different from each other from another vital consideration. Classical methods mostly attempt to scalarize multiple objectives and perform repeated applications to find a set of Pareto-optimal solutions. On the other hand, EMO methods attempt to find multiple Pareto-optimal solutions in a single simulation run.

However, there exist a few classical generating methods (stochastic and deterministic) which attempt to find multiple Pareto-optimal solutions in a single

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simulation run, very much similar to the way EMO methods work. In this paper, we present three such algorithms and provide simulation results on a number of two and three-objective optimization problems. We also compare their performance with an EMO methodology and unveil the problem classes where the classical generating methods are better and the problem classes where the EMO methods have their niche. The study reveals important insights about the working of the algorithms, which can be combined together in a hybrid manner to develop an algorithm even better than individual algorithms.

2 Classical Generating Methods

Although most classical generating multi-objective optimization methods use an iterative scalarization scheme of standard procedures such as weighted-sum or epsilon-constraint methods [8], we have found at least three generating methods which attempt to find multiple Pareto-optimal solutions in a single simulation run. In the following subsections, we describe these methods.

2.1 Schäffler's Stochastic Method (SSM)

A stochastic method for the solution of unconstrained multi-objective optimization problems was proposed by Schäffler et. al. [9] in 2002. The method is based on the solution of a set of stochastic differential equations. This method requires the objective functions to be twice continuously-differentiable. It may be used for the computation of all or a large number of Pareto-optimal solutions. In each iteration, a trace of non-dominated points is constructed by calculating at each point $\mathbf{x}$, a direction $(-q(\mathbf{x}))$ in the decision space which is a direction of descent for *all* objective functions. The direction of descent is obtained by solving a quadratic subproblem. The following initial value problem (IVP) for a multi-objective optimization problem is then set up:

$$\dot{\mathbf{x}} = -q(\mathbf{x}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0,$$

where x_0 is a starting point. The numerical solution of the above IVP gives a single point where the first-order weak Pareto-optimality conditions are fulfilled. After such a solution is obtained, a set of non-dominated solutions is obtained by perturbing it using a Brownian motion concept. The following stochastic differential equation is employed for this purpose:

$$d\mathbf{X}_t = -q(\mathbf{X}_t)d(t) + \varepsilon dB_t, \quad \mathbf{X}_0 = \mathbf{x}_0, \tag{1}$$

where $\varepsilon > 0$ and B_t is a n-dimensional Brownian motion having the following properties:

1. The expected value is zero,
2. The increments B_0, $(B_{t_1} - B_{t_0})$, $(B_{t_2} - B_{t_1})$ for every $t_0(= 0) < t_1 < t_2 < \ldots$ are stochastically independent, and
3. For every $s < t$, the increment $(B_s - B_t)$ is normally distributed with mean equal to zero and a variance equal to $(s - t)I_n$, where I_n is a n-dimensional identity matrix.