Selection, Drift, Recombination, and Mutation in Multiobjective Evolutionary Algorithms on Scalable MNK-Landscapes

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Abstract. This work focuses on the working principles, behavior, and performance of state of the art multiobjective evolutionary algorithms (MOEAs) on discrete search spaces by using MNK-Landscapes. Its motivation comes from the performance shown by NSGA-II and SPEA2 on epistatic problems, which suggest that simpler population-based multiobjective random one-bit climbers are by far superior. Adaptive evolution is a search process driven by selection, drift, mutation, and recombination over fitness landscapes. We group MOEAs features and organize our study around these four important and intertwined processes in order to understand better their effects and clarify the reasons to the poor performance shown by NSGA-II and SPEA2. This work also constitutes a valuable guide for the practitioner on how to set up its algorithm and gives useful insights on how to design more robust and efficient MOEAs.

1 Introduction

Epistasis in the context of evolutionary algorithms (EAs) describes nonlinearities in fitness functions due to changes in the values of interacting bits. Epistasis is recognized as an important factor that makes a problem difficult for optimization algorithms and its influence on the performance of single objective EAs is being increasingly investigated. Particularly, Kauffman's NK-Landscapes model of epistatic interactions [1] has been the center of several studies, both for the statistical properties of the generated landscapes and for their *EA-hardness*. See for example [2, 3, 4, 5] and there in. Studies on the behavior of single objective EAs on NK-Landscapes have proved useful to advance our understanding of EA's working principles and served to design robust and better algorithms [5].

Contrary to single objective EAs, studies concerning epistasis within the context of multiobjective evolutionary algorithms (MOEAs) are few and its effects still not well understood. Recently, Aguirre and Tanaka [6] have extended Kauffman's NK-Landscapes model of epistatic interactions to multiobjective MNK-Landscapes, giving insights into their properties in order to understand how the parameters of the landscapes relate to multiobjective concepts such as shape of the fronts, number of non-dominated fronts, number of non-dominated solutions,

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accessibility to the true Pareto front, correlation between and within fronts, and metrics. From a multiobjective random test problem generator standpoint [7], desirable features of MNK-Landscapes are that the problems are easy to construct and can scale to any number of objectives M, number of bits N, and number of epistatic interactions K, allowing the creation of sub-classes of combinatorial non-linear problems for discrete search spaces in which we can test the working principles of MOEAs in order to design better and more robust algorithms. Aguirre and Tanaka have also studied the behavior of multiobjective random one-bit climbers (moRBCs) [8] on MNK-Landscapes and have provided initial results on the performance of two well known representatives of the latest generation of elitist MOEAs [9], namely NSGA-II [10] and SPEA2 [11].

This work focuses on the working principles, behavior, and performance of state of the art MOEAs on discrete search spaces by using MNK-Landscapes. Its motivation comes from the performance shown by NSGA-II and SPEA2 on epistatic problems [9, 8], which suggest that simpler population-based moRBCs are by far superior. Adaptive evolution is a search process driven by selection, drift, mutation, and recombination over fitness landscapes [1]. We group MOEAs features and organize our study around these main processes. In most of the latest generation MOEAs [10, 12] selection incorporates elitism and it is biased by Pareto dominance and a diversity preserving strategy in objective space. Genetic operators vary according to whether the search space is continuous or discrete. In discrete search spaces, like MNK-Landscapes, recombination is usually implemented as one-point or two-point crossover and mutation as the standard bit flipping method. Some approaches also include specialized mutation operators to perform local search. In addition to these features explicit to the algorithm design, drift is also an important process that drives evolution and it is implicit to all stochastic algorithms working on finite small populations, although sometimes highly overlooked. In this paper we study the effects of these important and intertwined processes in order to understand them better, clarifying the reasons to the poor performance shown by NSGA-II and SPEA2. This work also constitutes a valuable guide for the practitioner on how to set up its algorithm and gives useful insights on how to design more robust and efficient MOEAs.

2 Multiobjective MNK-Landscapes

A multiobjective MNK-Landscape is defined as a vector function mapping binary strings into real numbers $\boldsymbol{f}(\cdot) = (f_1(\cdot),\, f_2(\cdot), \cdots, f_M(\cdot)) : \mathcal{B}^N \rightarrow \Re^M$, where M is the number of objectives, $f_i(\cdot)$ is the i-th objective function, $\mathcal{B} = \{0, 1\}$, and N is the bit string length. $\boldsymbol{K} = \{K_1, \cdots,\, K_M\}$ is a set of integers where K_i $(i = 1, 2, \cdots, M)$ is the number of bits in the string that epistatically interact with each bit in the i-th landscape. Each $f_i(\cdot)$ can be expressed as an average of N functions as follows

$$f_i(\boldsymbol{x}) = \frac{1}{N} \sum_{j=1}^{N} f_{i,j}(x_j, z_1^{(i,j)}, z_2^{(i,j)}, \cdots, z_{K_i}^{(i,j)}) \tag{1}$$