Fuzzy-Pareto-Dominance and Its Application in Evolutionary Multi-objective Optimization

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Abstract. This paper studies the fuzzification of the Pareto dominance relation and its application to the design of Evolutionary Multi-Objective Optimization algorithms. A generic ranking scheme is presented that assigns dominance degrees to any set of vectors in a scale-independent, non-symmetric and set-dependent manner. Based on such a ranking scheme, the vector fitness values of a population can be replaced by the computed ranking values (representing the "dominating strength" of an individual against all other individuals in the population) and used to perform standard single-objective genetic operators. The corresponding extension of the Standard Genetic Algorithm, so-called Fuzzy-Dominance-Driven GA (FDD-GA), will be presented as well. To verify the usefulness of such an approach, an analytic study of the Pareto-Box problem is provided, showing the characteristical parameters of a random search for the Pareto front in a unit hypercube in arbitrary dimension. The basic problem here is the loss of dominated points with increasing problem dimension, which can be successfully resolved by basing the search procedure on the fuzzy dominance degrees.

1 Introduction

In multiobjective optimization, the optimization goal is given by more than one objective to be extreme. Formally, given a domain as subset of R^n, there are assigned m functions $f_1(x_1,\ldots,x_n),\ldots,f_m(x_1,\ldots,x_n)$. Usually, there is not a single optimum but rather the so-called Pareto set of *non-dominated* solutions.

Evolutionary Computation (EC) has been shown to be a powerful technique for multi-objective optimization [1][2][3] (EMO - Evolutionary Multi-Objective Optimization). This biologically inspired methodology offers both flexibility in goal specification and good performance in multimodal, nonlinear search spaces.

If we want to solve a highly complex multi-objective optimization problem, we might select one of the best ranked evolutionary approaches reviewed in the literature, like NSGA-II [4] or SPEA2 [5] and hopefully start reaching good results quickly. However, all these algorithms need dominated individuals in the population, to perform the corresponding genetic operators. For a higher number of objectives, this might become a problem, since the probability of having a dominated individual in the population will rapidly go to zero.

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The need for a revision of the Pareto dominance relation for also handling a larger number of objectives was already pointed out in a few studies, esp. given by Farina and Amato [6]. There, we also find the suggestion to use fuzzy membership degrees for the degree of a point belonging to the Pareto set (so-called fuzzy optimality). Authors design their revised dominance measure in a way that the approach to the Pareto front can be registered more early in the search. The approach was shown to work successfully in the domain of more than three objectives. It came out that the use of fuzzy concepts is fruitful in this regard. However, the approach did not provide a direct means to formulate corresponding EMO algorithms. So, the limitation is that still only the relation between two points is considered.

In this paper, we are going to use concepts from fuzzy fusion theory to achieve a more far reaching goal. Instead of introducing only a degree of dominance for two points, we are going to fuse the mutual degrees of dominance within a set of points and assign a *ranking* value to each point within this set of points. The circumstances now allow for using these ranking values in the same fashion as single objectives, e.g. to rank individuals within a population and thus easily expanding the application field of a Standard Genetic Algorithm to the multi-objective domain. Note that this is not the same as reducing multiple objectives into a single objective (as it is done e.g. with the weighted sum aproach): the ranking values are only fixed for a point within a given set of points. Once the set changes, the ranking values can vary as well.

In [7] this approach was already shown to handle search problems with separated Pareto fronts. However, for a better understanding of such an approach, suitable test problems are also needed, what has been a long-termed research issue in EMO as well. Usually, the different algorithms are compared by measuring their performance indices in difficult test searches [1][2][3][8]. However, these problems are hard to analyze, and the characteristics of a random or genetic search can not be provided. This prevents us from keeping track of the populations dynamics unambiguously (as already stated by Coello in [2]). Thus, we also introduce an "easy" multi-objective test function that allows us to observe the search progress and that is yet easily scalable to higher number of objectives as well. The Pareto-Box Problem, which will be presented and studied in this paper as well, unifies these crucial properties. It will help us to know more about how the Pareto-front is searched for in EMO, and to measure the progress of the novel Fuzzy Pareto Dominance-Driven Genetic Algorithm (FDD-GA) approach in search problems with higher number of objectives.

In the following section, we will consider the Fuzzy-Pareto Dominance relation, and base an EMO algorithm on it in section 3. Then, section 4 defines the Pareto-Box problem and its analysis for random search. These results will be used in an exemplary manner to study the dynamics of EMOs in section 5. The paper ends with the conclusion and the reference.