Multi-objective Optimization of Problems with Epistemic Uncertainty

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Abstract. Multi-objective evolutionary algorithms (MOEAs) have proven to be a powerful tool for global optimization purposes of deterministic problem functions. Yet, in many real-world problems, uncertainty about the correctness of the system model and environmental factors does not allow to determine clear objective values. Stochastic sampling as applied in noisy EAs neglects that this so-called epistemic uncertainty is not an inherent property of the system and cannot be reduced by sampling methods. Therefore, some extensions for MOEAs to handle epistemic uncertainty in objective functions are proposed. The extensions are generic and applicable to most common MOEAs. A density measure for uncertain objectives is proposed to maintain diversity in the nondominated set. The approach is demonstrated to the reliability optimization problem, where uncertain component failure rates are usual and exhaustive tests are often not possible due to time and budget reasons.

1 Introduction

The traditional way to define optimization problems is to create a model of the system and state it to be exact and deterministic. Clearly defined decision values are mapped to likewise clearly defined, non-varying objective values.

Respecting the fact that nature doesn't adhere to determinism, stochastic optimization problems and their evolutionary solution methods emerged and gained importance [1]. Yet, the main part of this approaches still abide to the certainty of observed objectives. High sampling rates of a given decision value could simply reveal the underlying distribution of the random processes modelled by the system [2, 3]. MOEA approaches dealing with aleatory uncertainty are presented in [4] and [5].

Models of real systems are built without perfect knowledge of the system simulated. Often the objective values stay highly uncertain even if the real (aleatory) variance is minimal because of a fundamental lack of information about environmental factors or the system itself. In this case even infinitive sampling rates won't help as we simply don't know the distributions to sample from. This so-called epistemic uncertainty must not be ignored in the optimization process.

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Indeed, there is a trend in reliability science and other application areas to formulate models that incorporate and propagate epistemic uncertainties to the simulation outputs rather than generating sharp values. The results are very often only given as intervals or belief functions and thus a need for algorithms capable to handle this types of data is needed. Some approaches towards this issue can be found in [6] and [7].

This work is structured as followed. Section 2 introduces epistemic uncertainty modelling and its mathematical and computational representation. Section 3 discusses different possible extensions of common decision criteria for single and multi-objective evolutionary algorithms (MOEAs). Two ways of redefining the Pareto order over objective vectors and the order over one-dimensional objective functions are introduced. Section 4 shows a way to extend standard MOEAs to handle uncertain objectives in both selection and repository processes. The extension is generic and thus could be applied to most of the commonly used MOEAs. Section 5 proposes a niching strategy to prevent diversity among uncertain solutions. Section 6 shows the application of the proposed approach to the reliability design problem. Finally, some outlines and further research directions are proposed.

2 Belief, Plausibility and the Representation of Epistemic Uncertainty

2.1 Aleatory and Epistemic Uncertainty

There are at least two types of uncertainty that have to be distinguished because of their difference in origin, modelling and effects: Aleatory and epistemic uncertainty. Oberkampf et al. [8] defines aleatory uncertainty as the "inherent variation associated with the physical system or the environment under consideration". Aleatory uncertainty of a quantity can often be distinguished from other types of uncertainty by its characterization as a random value with known distribution. The exact value will change but is expected to follow the distribution. A simple example for aleatory uncertainty is the uncertainty about the outcome of a dice toss $X \in \{1, 2, 3, 4, 5, 6\}$. We are uncertain about the number we will receive, but we are sure that each of the numbers will occur with a probability $p(X = 1) \cdots p(X = 6) = 1/6$.

On the contrary, epistemic uncertainty describes not uncertainty about the outcome of some random event due to system variance but the uncertainty of the outcome due to "any lack of knowledge or information in any phase or activity of the modelling process"[8]. This shows the important difference between this two types of uncertainty. Epistemic uncertainty is not an inherent property of the system. A gain of information about the system or environmental factors can lead to a reduction of epistemic uncertainty. We now focus again on the dice example. Somebody told us that the dice is pronged and so we expect that the probability is limited as $p(X = 1) \dots p(X = 6) \in [1/12, 7/12]$. Of course, the dice would follow a distribution and if we would carry out an infinite number