1984-2004 – 20 Years of Multiobjective Metaheuristics. But What About the Solution of Combinatorial Problems with Multiple Objectives?

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Abstract. After 20 years of development of multiobjective metaheuristics the procedures for solving multiple objective combinatorial optimization problems are generally the result of a blend of evolutionary, neighborhood search, and problem dependent components. Indeed, even though the first procedures were direct adaptations of single objective metaheuristics inspired by evolutionary algorithms or neighborhood search algorithms, hybrid procedures have been introduced very quickly. This paper discusses hybridations found in the literature and mentions recently introduced metaheuristic principles.

1 Multiobjective Optimization

A multiobjective optimization problem is defined as

$$\min_{x\in X}(z_1(x),\ldots,z_p(x)), \tag{MOP}$$

where $X \subset \mathbb{R}^n$ is a feasible set in the *decision space*, and $z : \mathbb{R}^n \to \mathbb{R}^p$ is a vector valued objective function. By $Z = z(X) \subset \mathbb{R}^p$ we denote the image of the feasible set in the *objective space*. We consider optimal solutions of (MOP) in the sense of *efficiency* (or Pareto optimality), that is, a feasible solution $x \in X$ is called efficient if there does not exist $x' \in X$ such that $z(x') \leq z(x)$, i.e., $z_k(x') \leq z_k(x)$ for all $k = 1,\ldots,p$ and $z_j(x') < z_j(x)$ for some j. In other words, no solution is at least as good as x for all objectives, and strictly better for at least one.

Efficiency refers to solutions x in decision space. In terms of the objective space, with objective vectors $z(x) \in \mathbb{R}^p$ we use the notion of *non-dominance*: If x is an efficient solution then $z(x) = (z_1(x),\ldots,z_p(x))$ is a non-dominated

C. A. Coello Coello et al. (Eds.): EMO 2005, LNCS 3410, pp. 33–46, 2005.

vector (or point). The set of efficient solutions is X_E, the set of non-dominated vectors is Z_N. We may also refer to Z_N as the *non-dominated frontier* or the trade-off surface or the Pareto front. For $x^1, x^2 \in X$ we shall use the notation $x^1 \succ x^2$ if x^1 dominates x^2, i.e., if $z(x^1) \leq z(x^2)$.

In case of multiple feasible solutions $x, x' \in X$ mapping to the same non-dominated point $z(x) = z(x')$, the solutions are said to be *equivalent* [24]. A *complete set* X_E [24] is a set of efficient solutions such that all $x \in X \setminus X_E$ are either dominated or equivalent to at least one $x \in X_E$. I.e., for each nondominated point $z \in Z_N$ there exists at least one $x \in X_E$ such that $z(x) = z$. To solve a multiobjective optimization problem often means to find a complete set of efficient solutions. The computation of a set of efficient solutions is a major challenge of multiobjective optimization. But to precisely characterize the ability of an algorithm to solve an MOP the definition of complete set is refined as follows:

- [24] A *minimal complete set* X_{E_m} is a complete set without equivalent solutions. Any complete set contains a minimal complete set.
- [36] The *maximal complete set* X_{E_M} is a complete set including all equivalent solutions, i.e., all $x \in X \setminus X_{E_M}$ are dominated.

Multiobjective combinatorial optimization problems form a particular class of MOPs, which can be formulated as follows:

$$\min \left\{ Cx : Ax \geq b, x \in \mathbb{Z}^n \right\}. \tag{MOCO}$$

Here C is a $p \times n$ objective function matrix, where c_k denotes the k-th row of C. A is an $m \times n$ matrix of constraint coefficients and $b \in \mathbb{R}^m$. Usually the entries of C, A and b are integers. The feasible set $X = \{Ax \geq b, x \in \mathbb{Z}^n\}$ may describe a combinatorial structure such as, e.g., spanning trees of a graph, paths, matchings etc. We shall assume that X is a finite set. By $Z = CX$ we denote the image of X under C in $\mathbb{R}^p$.

2 Approximation Methods for MOCO

As in the single objective case, reasonable alternatives to exact methods for solving difficult MOCOs are approximation methods. An *approximation method* in a multiobjective optimization context is a method which finds either sets of locally potentially efficient solutions that are later merged to form a set of potentially efficient solutions – the approximation denoted by X_{PE} – or globally potentially efficient solutions according to the current approximation X_{PE}.

2.1 The Question of Quality of an Approximation

The quality of a solution of a combinatorial optimization problem can be estimated by comparing lower and upper bounds on the optimal objective function value. In multiobjective optimization the concept of bounds is not well developed. The best possible lower and upper bounds on values of all non-dominated points are given by the ideal and nadir point z^I and z^N defined by