A MOPSO Algorithm Based Exclusively on Pareto Dominance Concepts

Julio E. Alvarez-Benitez*, Richard M. Everson, and Jonathan E. Fieldsend**

Department of Computer Science, University of Exeter, UK

Abstract. In extending the Particle Swarm Optimisation methodology to multi-objective problems it is unclear how global guides for particles should be selected. Previous work has relied on metric information in objective space, although this is at variance with the notion of dominance which is used to assess the quality of solutions. Here we propose methods based exclusively on dominance for selecting guides from a non-dominated archive. The methods are evaluated on standard test problems and we find that probabilistic selection favouring archival particles that dominate few particles provides good convergence towards and coverage of the Pareto front. We demonstrate that the scheme is robust to changes in objective scaling. We propose and evaluate methods for confining particles to the feasible region, and find that allowing particles to explore regions close to the constraint boundaries is important to ensure convergence to the Pareto front.

1 Introduction

Evolutionary algorithms (EA) have been used since the mid-eighties to solve complex single and multi-objective optimisation problems (see, for example, [1, 2, 3]). More recently the Particle Swarm Optimisation (PSO) heuristic, inspired by the flocking and swarm behaviour of birds, insects, and fish schools has been successfully used for single objective optimisation, such as neural network training and non-linear function optimisation [4]. Briefly, PSO maintains a balance between exploration and exploitation in a population (swarm) of solutions by moving each solution (particle) towards both the global best solution located by the swarm so far and towards the best solution that the particular particle has so far located. The global best and personal best solutions are often called *guides*.

Since PSO and EA algorithms have structural similarities (such as the presence of a population searching for optima and information sharing between population members) it seems a natural progression to extend PSO to multi-objective problems (MOPSO). Some attempts in this direction have been made with promising results such as [5, 6, 7, 8, 9]. In the most recent heuristics the

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guides are selected from the set of non-dominated solutions found so far. However, in a multi-objective problem each of non-dominated solutions is a potential global guide and there are many ways of selecting a guide from among them for each particle in the swarm. Heuristics to date have relied on proximity in objective space to determine this selection, however the relative weightings of the objectives are *a priori* unknown and the use of metric information in objective space is at variance with the notion of dominance that is central to the definition of Pareto optimality. In this paper we propose and examine MOPSO heuristics based entirely on Pareto dominance concepts. The manner in which particles are constrained to lie within the search space can have a marked effect on the optimisation efficiency: the other central purpose of this paper is to propose and compare constraint methods.

We start by briefly reviewing basic definitions of multi-objective problems and Pareto concepts (section 2), after which we describe the single objective PSO methodology in section 3. The multi-objective PSO algorithm is presented in section 4, and we present and evaluate methods for selecting guides here. Techniques for confining particles to the feasible region are described and evaluated in section 5. Finally, conclusions are drawn in section 6.

2 Dominance and Pareto Optimality

In a multi-objective optimisation problem we seek to simultaneously extremise D objectives: $y_i = f_i(\mathbf{x})$, where $i = 1, \ldots, D$ and where each objective depends upon a vector $\mathbf{x}$ of K parameters or decision variables. The parameters may also be subject to the J constraints: $e_j(\mathbf{x}) \geq 0$ for $j = 1, \ldots, J$.

Without loss of generality it is assumed that these objectives are to be minimised, as such the problem can be stated as:

$$\text{minimise } \mathbf{y} = \mathbf{f}(\mathbf{x}) \equiv (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_D(\mathbf{x})) \tag{1}$$

$$\text{subject to } \mathbf{e}(\mathbf{x}) \equiv (e_1(\mathbf{x}), e_2(\mathbf{x}), \ldots, e_J(\mathbf{x})) \geq \mathbf{0}. \tag{2}$$

A decision vector $\mathbf{u}$ is said to *strictly dominate* another $\mathbf{v}$ (denoted $\mathbf{u} \prec \mathbf{v}$) if $f_i(\mathbf{u}) \leq f_i(\mathbf{v})\ \forall i = 1, \ldots, D$ and $f_i(\mathbf{u}) < f_i(\mathbf{v})$ for some i; less stringently $\mathbf{u}$ *weakly dominates* $\mathbf{v}$ (denoted $\mathbf{u} \preceq \mathbf{v}$) if $f_i(\mathbf{u}) \leq f_i(\mathbf{v})$ for all i. A set of decision vectors is said to be a *non-dominated set* if no member of the set is dominated by any other member. The *true* Pareto front, $\mathcal{P}$, is the non-dominated set of solutions which are not dominated by any feasible solution.

3 Particle Swarm Optimisation – PSO

The particle swarm optimisation method evolved from a simple simulation model of the movement of social groups such as birds and fish [4], in which it was observed that local interactions underlie the group behaviour and individual members of the group can profit from the discoveries and experiences of other