# A Multi-objective Tabu Search Algorithm for Constrained Optimisation Problems

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**Abstract.** Real-world engineering optimisation problems are typically multi-objective and highly constrained, and constraints may be both costly to evaluate and binary in nature. In addition, objective functions may be computationally expensive and, in the commercial design cycle, there is a premium placed on rapid initial progress in the optimisation run. In these circumstances, evolutionary algorithms may not be the best choice; we have developed a multi-objective Tabu Search algorithm, designed to perform well under these conditions. Here we present the algorithm along with the constraint handling approach, and test it on a number of benchmark constrained test problems. In addition, we perform a parametric study on a variety of unconstrained test problems in order to determine the optimal parameter settings. Our algorithm performs well compared to a leading multi-objective Genetic Algorithm, and we find that its performance is robust to parameter settings.

## 1 Introduction

Real-world optimisation problems have a number of characteristics which must be taken into account when developing optimisation algorithms. Real-world problems are typically multi-objective; trade-offs between risk and reward, and cost and benefit exist at a fundamental level throughout the natural world and are a deep-seated part of human consciousness. These trade-offs carry over directly to the business world, and thus into any form of design activity. Any optimisation method which is to have any serious benefit to the design process must be able to handle multiple objectives.

Real-world problems also tend to be highly constrained. The nature of these constraints and their effect on the optimisation landscape varies from problem to problem. However, optimisation problems in a number of fields have constraints with similar characteristics, and this is discussed further in Section 1.1 below. The optimisation landscape – regions of feasible, highly constrained design space and the variations of objective function values within that space – is strongly influenced by the parameterisation scheme, for any one given problem. Good parameterisation schemes for aerodynamic shape optimisation problems – the particular focus of our work – as shown by Harvey [1], Kellar [2] and Gaiddon

C. A. Coello Coello et al. (Eds.): EMO 2005, LNCS 3410, pp. 490–504, 2005.

*et al.* [3], tend to produce optimisation landscapes that are highly constrained, have many variables, and many local minima. Thus, the optimisation algorithm must be chosen to perform well in these circumstances [4].

These characteristics quickly rule out the use of traditional gradient-based optimisation methods: notwithstanding their requirement of gradient information (which may be difficult, expensive, or impossible to obtain), these algorithms perform poorly in problems which are highly constrained and contain local minima. Harvey [1] tested a number of meta-heuristic methods on a representative aerodynamic design optimisation problem and found Tabu Search (TS) to be superior to the Genetic Algorithm (GA) and Simulated Annealing (SA) methods.

Numerous of multi-objective GAs exist [5]. Similarly, multi-objective SA methods have been developed [6]. However, despite its popularity in single-objective optimisation problems, very few attempts have been made at developing a multi-objective version of TS. Jones [7] reviewed the literature on multi-objective meta-heuristics and found only 6% of 124 papers concerned with TS.

Given that it may well perform better than a GA or SA method (assuming Harvey's results carry over into multi-objective optimisation) on aerodynamic design optimisation problems, and there is a strong real-world requirement to perform multi-objective optimisation, there appears to be both a need and an opportunity to develop a new multi-objective TS algorithm.

### 1.1 Constraint Handling

It is important that the constraint handling method of an optimisation algorithm is able to deal with constraints which are binary, as happens in a number of real-world engineering problems. The constraint handling in many multi-objective evolutionary algorithms requires the ability to assign some kind of constraint violation distance to points in design space which violate constraints, and points are then ranked accordingly [5]. In the presence of binary constraints, such an approach cannot be used.

Such constraints occur typically in shape optimisation problems, especially when the cost function is evaluated using a finite difference type of method (including finite element and finite volume methods) which solves a system of equations over a finite mesh. The constraints for these problems arise from three sources, amongst others:

1. *Geometric considerations.* The parameterisation scheme may give rise to shapes which are physically impossible (*i.e.* negative volumes). Conceptually, a distance measure in design space may be formulated by considering an offset vector $\Delta\bar{x}$ which can be added to the design vector $\bar{x}$ to make the design feasible; in practice this may be too costly due to the interdependence between design variables.
2. *Mesh considerations.* Given a valid geometry, it may be impossible to fit a mesh that satisfies certain criteria relevant to the numerical solution of the problem (*i.e.* skewed cells). Again, finding an offset vector $\Delta\bar{x}$ is conceptually possible, but is in practice even more costly than finding a $\Delta\bar{x}$ that satisfies just the geometric considerations.