An EMO Algorithm Using the Hypervolume Measure as Selection Criterion

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Abstract. The hypervolume measure is one of the most frequently applied measures for comparing the results of evolutionary multiobjective optimization algorithms (EMOA). The idea to use this measure for selection is self-evident. A steady-state EMOA will be devised, that combines concepts of non-dominated sorting with a selection operator based on the hypervolume measure. The algorithm computes a well distributed set of solutions with bounded size thereby focussing on interesting regions of the Pareto front(s). By means of standard benchmark problems the algorithm will be compared to other well established EMOA. The results show that our new algorithm achieves good convergence to the Pareto front and outperforms standard methods in the hypervolume covered.
We also studied the applicability of the new approach in the important field of design optimization. In order to reduce the number of time consuming precise function evaluations, the algorithm will be supported by approximate function evaluations based on Kriging metamodels. First results on an airfoil redesign problem indicate a good performance of this approach, especially if the computation of a small, bounded number of well-distributed solutions is desired.

1 Introduction

Pareto optimization [1, 2] has become a well established technique for detecting interesting solution candidates for multiobjective optimization problems. It enables the decision maker to filter efficient solutions and to discover trade-offs between opposing objectives among these solutions. Provided a set of objective functions $f_{1,\dots,n} : \mathbb{S} \to \mathbb{R}$ defined on some search space $\mathbb{S}$ to be minimized, in Pareto optimization the aim is to detect the *Pareto-optimal set* $M = \{\mathbf{x} \in \mathbb{S} | \nexists \mathbf{x}' \in \mathbb{S} : \mathbf{x}' \prec \mathbf{x}\}$, or at least a good approximation to this set.

In practice, the decision maker wishes to evaluate only a limited number of Pareto-optimal solutions. This is due to the limited amount of time for examining the applicability of the solutions to be realized in practice. Typically these

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solutions should include extremal solutions as well as solutions that are located in parts of the solution space, where balanced trade-offs can be found.

A measure for the quality of a non-dominated set is the hypervolume measure or $\mathcal{S}$ metric [3]. Until now, research mainly focussed on two approaches to utilize the $\mathcal{S}$ metric for multiobjective optimization: Fleischer [4] suggested to recast the multiobjective optimization problem to a single objective one by maximizing the $\mathcal{S}$ metric of a finite set of non-dominated points. Knowles et al. utilized the $\mathcal{S}$ metric within an archiving strategy for EMOA [5, 6].

Going one step further, our aim was to construct an algorithm in which the $\mathcal{S}$ metric governs the selection operator of an EMOA in order to find a set of solutions well distributed on the Pareto front. The basic idea of this EMOA is to integrate new points in the population, if replacing a member increases the hypervolume covered by the population. Moreover, we aimed at an algorithm that can easily be parallelized and is simple and transparent. It should be extendable by problem specific features, like approximate function evaluations. Thus, a steady-state $(\mu + 1)$-EMOA, the so-called *$\mathcal{S}$ metric selection EMOA (SMS-EMOA)*, is proposed.

Notice that in contrast to Knowles et al. [6], we do not evaluate an archiving operator solely, but the dynamics of a complete EMOA based on $\mathcal{S}$ metric selection. In our opinion, the design of an EA suitable for a given problem or a series of test problems is a multiobjective task again. This way we look at archiving strategies as only one component of the whole EMOA.

The article is structured as follows: The hypervolume or $\mathcal{S}$ metric that is used in the selection of our algorithm is discussed first (section 2). Afterwards, the integration in an EMOA as well as some features are described (section 3). Section 4 deals with the performance on several test problems whereas the results achieved on a real world design problem are the topic of section 5, including results with approximate function evaluations. In particular, the coupling of our method to a metamodel assisted fitness function approximation tool is presented here. We close with a summary and an outlook to implied future tasks (section 6).

2 The Hypervolume Measure

The hypervolume measure or S metric was originally proposed by Zitzler and Thiele [3], who called it the *size of the space covered* or *size of dominated space*. Coello Coello, Van Veldhuizen and Lamont [2] described it as the Lebesgue measure Λ of the union of hypercubes a_i defined by a non-dominated point m_i and a reference point x_{ref}:

$$\mathcal{S}(M) := \Lambda(\{\bigcup_i a_i | m_i \in M\}) = \Lambda(\bigcup_{m \in M} \{x | m \prec x \prec x_{ref}\}). \tag{1}$$

Zitzler and Thiele note that this measure prefers convex regions to non-convex ones [3]. A major drawback was the computational time for recursively calculating the values of $\mathcal{S}$. Knowles and Corne [5] estimated $O(k^{n+1})$ with k being the number of solutions in the Pareto set and n being the number