Finding Pareto-Optimal Set by Merging Attractors for a Bi-objective Traveling Salesmen Problem

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Abstract. This paper presents a new search procedure to tackle multi-objective traveling salesman problem (TSP). This procedure constructs the solution attractor for each of the objectives respectively. Each attractor contains the best solutions found for the corresponding objective. Then, these attractors are merged to find the Pareto-optimal solutions. The goal of this procedure is not only to generate a set of Pareto-optimal solutions, but also to provide the information about these solutions that will allow a decision-maker to choose a good compromise solution.

1 Introduction

A multi-objective optimization seeks to optimize a vector of non-commensurable and often competing objectives. In other words, we whish to find a set of values for the decision variables that optimizes a set of objective functions. The general multi-objective combinatorial optimization problem can be formulated as:

$$optimize \quad f(x) = \begin{Bmatrix} f_1(x) = z_1 \\ f_2(x) = z_2 \\ \vdots \\ f_k(x) = z_k \end{Bmatrix} = z \in Z \tag{1}$$

$$subject to \quad x = (x_1, x_2, \ldots, x_n) \in X$$

where x is the decision vector, or *solution*, and $X \in \Re^n$ is the n-dimensional *decision space*, consisting of a finite set of feasible solutions. The objective function $f(x)$ maps x into $Z \in \Re^k$, the k-dimensional *objective space*, where k is the number of objectives. Whereas a single-objective problem is typically studied in decision space, multi-objective optimization is mostly studied in objective space. The image of a solution in the objective space is a point, $z = [z_1, z_2, \ldots, z_k]$. A point, z, is attainable if there exists a solution $x \in X$ such that $z = f(x)$. The set of all attainable points is denoted as Z. The ideal objective vector z^* is defined as $z^* = [\text{opt} f_1(x), \text{opt} f_2(x), \ldots, \text{opt} f_k(x)]$, which is obtained by optimizing each of the objective functions individually. Normally, the ideal objective vector is not attainable because of the conflict among the objectives. Therefore, there will not exist a single optimal solution to the multi-objective

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combinatorial problem. Instead, we must look for "trade-off" solutions when dealing with a multi-objective optimization problem.

Objective vectors are compared according to the concept of Pareto-optimality and dominance relation. A partial ordering can be applied to solutions to the problem by the dominance criterion. A solution $x^a \in X$ is said to dominate a solution $x^b \in X$ if x^a is superior or equal in all objectives and at least superior in one objective. Mathematically, the concept of *Pareto optimality* is as follows [21]: assume, without loss of generality, a minimization problem, and consider two decision vectors, x^a, $x^b \in X$, then x^a is said to dominate x^b (often written as $x^a \succ x^b$) if and only if

$$\begin{aligned} &\forall i \in \{1,2,...,k\}: f_i(x^a) \leq f_i(x^b) \quad \wedge \\ &\exists j \in \{1,2,\ldots,k\}: f_j(x^a) < f_j(x^b) \end{aligned} \tag{2}$$

The solution x^a is said to be indifferent to a solution x^b, if neither solution is dominating the other one. When no a priori preference is defined among the objectives, dominance is the only way to determine if one solution performs better than the other does. The concept of Pareto optimality almost gives us a set of solutions called the *Pareto-optimal set*. The solutions in the Pareto-optimal set are also called *nondominated*, characterized by the fact that starting from a solution within the set, one objective can only be improved at the expense of at least one other objective being deteriorated. The curve formed by joining the Pareto-optimal solutions is known as a *Pareto-optimal front*. The goal of solving multi-objective problem is to find the Pareto-optimal set for the decision-maker to choose the most preferred solution. A solution selected by the decision-maker always represents a compromise between the different objectives.

The bounds on the Pareto-optimal set in the objective space can be defined by the ideal point and the nadir point [16]. The ideal objective vector, z^*, denotes an array of the lower bound of all objective functions. For each of the k conflicting objectives, there exists one different optimal solution. An objective vector constructed with these individual optimal objective values constitutes the ideal objective vector z^*. In general, the ideal objective vector corresponds to a non-existent solution. This is because the optimal solution for each objective function need not be the same solution. The nadir objective vector, z^{nad}, represents the upper bounds of each objective in the entire Pareto-optimal set.

The problem of finding the true Pareto-optimal set is NP-hard [5]. Thus, the goal of the multi-objective combinatorial optimization is to approximate the Pareto-optimal set. Over the years, the work of a considerable number of researchers has produced an important number of techniques to deal with multi-objective optimization problems [4], [7], [16], [22].

The TSP is the most well-known of all NP-hard combinatorial optimization problems. Multi-objective TSP is even harder than its corresponding single-objective version. Some researches have specifically treated the multi-objective TSP. Fischer and Richter [8] used a branch and bound approach to solve a TSP with two (sum) criteria. Gupta and Warburton [9] used the 2- and 3-opt heuristics for the max-ordering TSP. Sigal [20] proposed a decomposition approach for solving the TSP with respect to the two criteria of the route length and bottlenecking, where both objectives are obtained from the same cost matrix. Tung [23] used a branch and bound method with a