Multiobjective EA Approach for Improved Quality of Solutions for Spanning Tree Problem

Rajeev Kumar, P.K. Singh, and P.P. Chakrabarti

Department of Computer Science and Engineering,
Indian Institute of Technology Kharagpur,
Kharagpur, WB 721 302, India
{rkumar, pksingh, ppchak}@cse.iitkgp.ernet.in

Abstract. The problem of computing spanning trees along with specific constraints is mostly NP-hard. Many approximation and stochastic algorithms which yield a single solution, have been proposed. In this paper, we formulate the generic multi-objective spanning tree (MOST) problem and consider edge-cost and diameter as the two objectives. Since the problem is hard, and the Pareto-front is unknown, the main issue in such problem-instances is how to assess the convergence. We use a multiobjective evolutionary algorithm (MOEA) that produces diverse solutions without needing a priori knowledge of the solution space, and generate solutions from multiple tribes in order to assess movement of the solution front. Since no experimental results are available for MOST, we consider three well known diameter-constrained minimum spanning tree (dc-MST) algorithms including randomized greedy heuristics (RGH) which represents the current state of the art on the dc-MST, and modify them to yield a (near-) optimal solution-fronts. We quantify the obtained solution fronts for comparison. We observe that MOEA provides superior solutions in the entire-range of the Pareto-front, which none of the existing algorithms could individually do.

1 Introduction

Computing a minimum spanning tree (MST) from a connected graph is a well-studied problem and many fast algorithms and analytical analyses are available [1, 2, 3, 4, 5, 6, 7, 8]. However, many real-life network optimization problems require the spanning tree to satisfy additional constraints along with minimum edge-cost. For example, communication network design problem for multicast routing of multimedia communication requires constructing a minimal cost spanning/Steiner tree with given constraints on diameter. VLSI circuit design problems aim at finding minimum cost spanning/Steiner trees given delay bound constraints on source-sink connections. Analogously, there exists the problem of degree/diameter-constrained minimum cost networks in many other engineering applications too (see [3] and the references therein).

Many such MST problem instances having a bound on the degree, a bound on the diameter, capacitated trees or bounds for two parameters to be satisfied simultaneously are listed in [3]. Finding spanning trees of sufficient generality and of minimal cost subject to satisfaction of additional constraints is often NP-hard [3, 4]. Many such design problems have been attempted and approximate solutions obtained using heuristics.

C. A. Coello Coello et al. (Eds.): EMO 2005, LNCS 3410, pp. 811–825, 2005.

For example, the research groups of Deo et al. [5, 6, 7, 8] and Ravi et al. [3, 4] have presented approximation algorithms by optimizing one criterion subject to a budget on the other. In recent years, evolutionary algorithms (EAs) have emerged as powerful tools to approximate solutions of such NP-hard problems. For example, Raidl & Julstrom [9, 10] and Knowles & Corne [11, 12] attempted to solve diameter and degree constrained minimum spanning tree problems, respectively using EAs. All such approximation and evolutionary algorithms yield a *single* optimized solution subject to satisfaction of the constraint(s). Moreover, researchers have demonstrated superiority of one algorithm over other algorithms for a *particular* value of a constraint and did not assess the performance over entire range of the values.

We argue that such constrained MST problems are essentially multiobjective in nature. A multiobjective optimizer yields a set of all representative equivalent and diverse solutions; the set of all optimal solutions is the Pareto-front. Secondly, extending this constraint-optimization approach to multi-criteria problems (involving two or more than two objectives/constraints) the techniques require improving upon more than one constraints. Thirdly and more importantly, such approaches may not yield all the representative optimal solutions. For example, most conventional approaches to solve network design problems start with a minimum spanning tree (MST), and thus effectively minimize the cost. With some variations induced by ϵ-constraint method, most other solutions obtained are located near the minimal-cost region of the Pareto-front, and thus do not form the complete (approximated) Pareto-front.

In this work, we try to overcome the disadvantages of conventional techniques and single objective EAs. We use multiobjective EA to obtain a (near-optimal) Pareto-front. For a wide-ranging review, a critical analysis of evolutionary approaches to multiobjective optimization and many implementations of multiobjective EAs, see [13, 14] for algorithms and implementations, and [15] for various applications.

We use Pareto Converging Genetic Algorithm (PCGA) [16] which has been demonstrated to work effectively across complex problems and achieves diversity without needing *a priori* knowledge of the solution space. PCGA excludes any explicit mechanism to preserve diversity and allows a natural selection process to maintain diversity. Thus multiple, equally good solutions to the problem, are provided. Another major challenge to solving unknown problems is how to ensure convergence. Some multiobjective problems have a tendency to get stuck at local Pareto-front [16], therefore, we generate solutions using multiple tribes and merge them to ensure convergence. PCGA assesses convergence to the Pareto-front which, by definition, is unknown in most real search problems of multi-dimensionality, by use of rank-histograms [17]. We consider, without loss of generality, edge-cost and tree-diameter as the two objectives to be minimized, though the framework presented here is generic enough to include any number of objectives to be optimized. Initial results of this work were presented in other conferences [18, 19]. In this paper, we extend the work for larger problem instances, present a systematic approach to assess the convergence, and compare qualitatively and quantitatively the obtained solution-fronts from three well-known techniques, namely, One-Time-Tree Construction (OTTC) [7], Iterative Refinement (IR) [7], and Randomized Greedy Heuristics (RGH) [9] algorithms.