# Triadic Concept Graphs and Their Conceptual Contents

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**Abstract.** *Concept graphs* are mathematizations of asserting propositions consisting of (dyadic) concepts and objects. In order to take conditions or modalities in consideration *triadic concept graphs* are introduced as a straightforward generalization based on the basic notions of *Triadic Concept Analysis*. Then concept implications are discussed and *conceptual contents* of triadic concept graphs are introduced. It turns out that this approach can be reduced to a dyadic view; and the Basic Theorem on Conceptual Contents is obtained as a consequence of that. Finally, triadic concept graphs are generalized by introducing a subdivision, i.e. concept graphs with a more complex "rhetoric structure" are considered.

## Contents



## 1 Triadic Concept Graphs

In this section we will recall some of the basic notions of *Contextual Concept and Judgment Logic* ([Wi04]). For an overview about *Contextual Logic*, its philosophical roots and its aims see [Wi00]. Here, judgments are understood as assertional combinations of concepts. Since the semantics should be based on *Formal Concept Analysis*, formal contexts are extended to *power context families* in order to express $k$-ary relation concepts (cf. [Wi02]):

**Definition 1.** *A sequence* $\vec{\mathbb{K}} := (\mathbb{K}_0, \mathbb{K}_1, \mathbb{K}_2, \ldots)$ *of contexts* $\mathbb{K}_k := (G_k, M_k, I_k)$ *with* $G_k \subseteq (G_0)^k$ *is called a* power context family. *The concepts of the contexts* $\mathbb{K}_k$ $(k \geq 1)$ *are called* relation concepts *since their extents are relations on the set* $G_0$. □

The elementary judgments which we consider have the form: An object (or a sequence of objects) is in the extent of a concept (or relation concept). Concept

B. Ganter and R. Godin (Eds.): ICFCA 2005, LNAI 3403, pp. 285–298, 2005.

graphs represent asserting combinations of such elementary judgments; to make it readable we use *relational graphs* as "rhetoric structure".

**Definition 2.** *A* relational graph *is a structure* $(V, E, \nu)$ *of two sets* $V$ *and* $E$ *and a map* $\nu : E \to \bigcup_{k=1,2,\ldots} V^k$*. The elements of* $V$ *and* $E$ *are called* vertices *and* edges*, respectively. An edge* $e$ *linked by* $\nu$ *with* $k$ *vertices is called* $k$-ary *– in symbols:* $|e| := k$*. Furthermore let* $E^{(k)} := \{e \in E \mid |e| = k\}$ *and* $E^{(0)} := V$*.* $\square$

An example of a relational graph is depicted in Figure 1. There are two vertices (the rectangles) and a 2-ary edge (the ellipse); the map $\nu$ is represented by the lines (with the numbers) between them. Now, we are able to define concept graphs of power context families (cf. [Wi02]):

**Definition 3.** *Let* $\vec{\mathbb{K}} := (\mathbb{K}_0, \mathbb{K}_1, \mathbb{K}_2, \ldots)$ *be a power context family. A structure* $\mathfrak{G} := (V, E, \nu, \kappa, \rho)$ *is called a* concept graph of $\vec{\mathbb{K}}$ *if:*

- $(V, E, \nu)$ *is a relational graph,*
- $\kappa : V \cup E \to \bigcup_{k=0,1,2,\ldots} \mathfrak{B}(\mathbb{K}_k)$ *such that* $\kappa(E^{(k)}) \subseteq \mathfrak{B}(\mathbb{K}_k)$ *for all* $k \geq 0$*,*
- $\rho : V \to \mathfrak{P}(G_0) \setminus \{\emptyset\}$ *is a map such that*
  - $v \in V \Rightarrow \rho(v) \subseteq Ext(\kappa(v))$*,*
  - $e \in E$ *with* $\nu(e) = (v_1, \ldots, v_k) \Rightarrow \rho(e) := \rho(v_1) \times \cdots \times \rho(v_k) \subseteq Ext(\kappa(e))$*.*

$\square$

A concept graph can be understood as a relational graph with entries of a power context family. The concept graph in Figure 1 represents the expression: "The woman Ruth skies with the man Peter." The underlying power context family is given by Fig. 2.


**Fig. 1.** Dyadic concept graph

| $\mathbb{K}_0$ | woman | man |
|---|---|---|
| Ruth | × | |
| Peter | | × |
| Henry | | × |
| John | | × |

| $\mathbb{K}_2$ | ski with | go for a walk with | go swimming with |
|---|---|---|---|
| (Ruth, Peter) | × | | |
| (Ruth, Henry) | | × | |
| (Ruth, John) | | | |

**Fig. 2.** Dyadic power context family

In order to take conditions or modalities in our considerations we recall the basic notions of *Triadic Concept Analysis*. For a motivating discussion of the following definitions of *triadic contexts* and *triadic concepts* see [LW95].