Protoconceptual Contents and Implications

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Abstract. The development of a mathematical model for judgments understood as compositions of concepts and relations has been an important branch of research in recent years. It led to the definitions of *concept* and *protoconcept graphs* which are based on information contained in a *power context family*, where incidence relations between objects (or tuples of objects) and attributes are stored.

A theory of the information those graphs represent (called *conceptual content*) has been developed for concept graphs in [PW99] and [Wi03]. In [HK04], an extension of this theory to protoconcept graphs not considering object implications (as it is done for concept graphs) has been established. The first part of this paper concentrates on the investigation of the protoconceptual content of protoconcept graphs respecting both protoconceptual and object implications.

The second part compares the different structures of conceptual and protoconceptual contents of a given power context family, showing how more background information (using object implications and concepts instead of protoconcepts) reduces the number of possible contents.

The third and final part analyzes how the different approaches can be generalized. Here we will concentrate on the (generalized) conceptual content of a formal context.

In each part an *information context* will be defined, which provides an accessible representation of the lattice of (proto-)conceptual closures.

1 Introduction

Concept and protoconcept graphs are means to graphically represent relational information, i. e. that an object or a tuple of objects is in a certain relation (or is not for the case of protoconcept graphs). While the original reason for the development of the theory of concept and protoconcept graphs is the aspect of visualization and the support of rational communication (see [Wi97, Wi00b, DK03]), we will concentrate here on the aspect of (proto-)conceptual content.

Different (proto-)concept graphs may represent the same information, resulting in a twofold problem: on the one hand, the problem of finding in the set of graphs representing the same information one that is suited for the purpose

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of rational communication; and, on the other hand, analyzing the structure of representable information, i.e. of the (proto-)conceptual contents generated by the graphs.

As seen in [PW99] the conceptual contents of concept graphs form a complete lattice, and this remains true for the protoconceptual contents of protoconcept graphs (see also [HK04]). For the case of conceptual contents Wille extended in [Wi03] the notion of conceptual content and found a formal context having the conceptual contents as extents. We adapted this approach in [HK04] with a restricted notion of content – this restriction will be removed with the results in Section 3.

As concepts are special protoconcepts and concept graphs are special protoconcept graphs (up to isomorphism), we are interested in the relationship of the resulting structures. Starting from protoconceptual contents without object implications and gradually adding background information up to the case of conceptual content with object implications we find that the set of possible contents is reduced. We can identify the structure of contents of the case with more background information as substructure of the one with less background information. This will be shown in Section 4.

All approaches, those for concept graphs and also the ones in [HK04] and in this paper have much in common. In Section 5 an approach is presented, which abstracts from the concrete cases allowing us to adapt the information context for variants of implications for objects or (proto-)concepts. Section 6 ends the paper with a conclusion.

2 Basic Definitions

In this section, we shortly recall the definitions for protoconcepts, protoconcept graphs and conceptual contents. The adaption of the definition of conceptual contents to protoconceptual contents (respecting object implications) will be discussed in the following section. For a more detailed introduction to these topics we refer the reader to [Wi00, Wi03] and [HK04].

A *protoconcept* (cf. [Wi00]) of $\mathbb{K} := (G, M, I)$ is defined as a pair (A, B) with $A \subseteq G$ and $B \subseteq M$ such that $A^I = B^{II}$ (which is equivalent to $A^{II} = B^I$). The set $\mathfrak{P}(\mathbb{K})$ of all protoconcepts of $\mathbb{K}$ is structured by the *generalization order* $\sqsubseteq$, defined by

$$(A_1, B_1) \sqsubseteq (A_2, B_2) :\Leftrightarrow A_1 \subseteq A_2 \text{ and } B_1 \supseteq B_2,$$

and by the following operations:

$$\begin{array}{rcl} (A_1, B_1) \sqcap (A_2, B_2) & := & (A_1 \cap A_2, (A_1 \cap A_2)^I) \\ (A_1, B_1) \sqcup (A_2, B_2) & := & ((B_1 \cap B_2)^I, B_1 \cap B_2) \end{array}$$

$$\begin{array}{ccc} \neg(A, B) := (G \backslash A, (G \backslash A)^I) & & (A, B) := ((M \setminus B)^I, M \setminus B) \\ \top := (G, \emptyset) & \text{and} & \bot := (\emptyset, M). \end{array}$$

The set $\mathfrak{P}(\mathbb{K})$ together with the operations $\sqcap, \sqcup, \neg, \lrcorner, \top$ and $\bot$ is called the *algebra of protoconcepts* of $\mathbb{K}$ and denoted by $\underline{\mathfrak{P}}(\mathbb{K})$. The operations are called