2 Introduction to the Physics of Gases

In physical textbooks and especially in thermodynamic textbooks there are more or less detailed explanations with the most important equations. For a more in-depth study the monograph from (Atkins 2006) is recommended as detailed as well as the German handbook from (Messer Griesheim 1989) and the monograph from (Hering et al. 1999).

2.1 The Ideal Gas

Since it has been possible to examine gases using physical methods two questions have always been of interest:

– How do the pressure $p$ and the volume $V$ of an enclosed quantity of gas behave at a constant temperature $T$ (isothermal state change)?
– How does the volume of a gas behave at changing temperatures but constant pressure (isobaric state change)?

The answers can be found in Boyle-Mariotte’s Law (17th century) according to the equation.

$$p \cdot V = \text{constant} = k \quad (2.1-1)$$

with $k$ as a constant and in Gay-Lussac’s Law (around 1800)

$$V_{T_1} = V_{T_0} \cdot (1 + \alpha \cdot \Delta T_{i-0}) \quad (2.1-2)$$

with a constant $p$ for the states 0 and 1, and with the temperature difference $\Delta T_{i-0} = T_1 - T_0$ and $\alpha$ as the (spatial) expansion coefficient of gas. $\alpha$ was determined by experiment and is found to be in a borderline case $p \to 0$ as $1/(273.15)$ K. A gas with this borderline case is described as ideal.

All known gases show no ideal behaviour, rather, they are so called “real gases”. The above mentioned Eqs. for the ideal gas can be used, if the temperatures are clearly above the melting point or the triple point and/or the pressure or the differences in pressure are small. The monoatomic He as the lightest inert gas comes closest to this ideal behaviour which is why it is used in gas thermometers.

The analogous equation (2.1-2) also applies to the behaviour of pressure at a constant geometric volume (isometric state change).

$$p_{T_1} = p_{T_0}(1 + \alpha \cdot \Delta T_{i-0}) \quad (2.1-3)$$
Example E2.1-1: Calculating pressure with varying temperature.

At 15°C the pressure of the gas cylinder is determined to 198 bar gauge-pressure, thus the absolute pressure is 199 bar. A temperature of 35°C is expected, consequently \( \Delta T = 20°C \). By approximation

\[
\rho_{35} = 199(1 + \frac{1}{273.15} \cdot 20) = 213.6 \text{[bar]}
\]

The equations (2.1-2 and -3) assume a very simple form when \( V_{T_0} \) is equal to \( V_0 \) for 0°C. Taking \( \alpha = (273.15 \text{ K})^{-1} = (T_{\text{stand}})^{-1} \) into consideration one obtain

\[
V_1 = V_0 \frac{T_1}{T_{\text{stand}}} \quad \text{and} \quad p_1 = p_0 \frac{T_1}{T_{\text{stand}}}
\]

with \( T \) in K or more generally

\[
\frac{V}{T} = k_1 \quad \text{and} \quad \frac{p}{T} = k_2
\]

2.1.1 State Equations

The combination of Boyle-Mariotte’s Law and Gay-Lussac’s Law produces the state equation for the ideal gas.

\[
\frac{p \cdot V}{T} = k
\]

where \( k \) indicates a specific constant in each case. With a fixed but arbitrary mass \( M \) of an ideal gas for two states 1 and 2 and the standard condition this equation is therefore valid.

\[
\frac{p_1 \cdot V_1}{T_1} = \frac{p_2 \cdot V_2}{T_2} = \frac{p_{\text{stand}} \cdot V_{T-\text{stand}}}{T_{\text{stand}}}
\]

with \( V_{T-\text{stand}} \) the volumes for \( V_1 \) and \( V_2 \) under standard conditions. By introducing the experimentally measured density of the real gas \( G \) at standard conditions \( \phi_{G,\text{STP}} \) and the specific gas constant \( R_G \) a simplification is obtained.

\[
V_{G,T-\text{stand}} = \frac{M_G}{\phi_{G,\text{STP}}} = V_{G,\text{STP}}
\]

\[
\frac{p \cdot V}{T} = \frac{M_G \cdot p_{\text{stand}}}{T_{\text{stand}} \cdot \phi_{G,\text{STP}}}
\]

\[
R_G = \frac{p_{G,\text{stand}}}{T_{\text{stand}} \cdot \phi_{G,\text{STP}}}
\]

\[
p \cdot V = M \cdot R_G \cdot T
\]