Solution of one mixed problem for equation of relaxational filtration by Monte Carlo methods

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1 Statement of problem

The model of filtration is considered according to the elementary nonequilibrium law in elastic surroundings. The relaxation cores of the filtration law $F(t)$ and the compressibility law $\Phi(t)$ have the following form:

$$F(t) = \frac{\mu}{\kappa} \left\{ t + (\tau_w - \tau_p) \left[ 1 - \exp\left( \frac{-t}{\tau_p} \right) \right] \right\} \eta(t), \quad \Phi(t) = \rho_0 \beta \eta(t),$$

and the model is described by the following system of equations:

$$\chi \Delta \left( p(t, x) + \tau_p \frac{\partial p(t, x)}{\partial t} \right) = \frac{\partial}{\partial t} \left( p + \tau_w \frac{\partial p(t, x)}{\partial t} \right), \quad (1)$$

$$-\frac{\kappa}{\mu} \text{grad} \left( p + \tau_p \frac{\partial p(t, x)}{\partial t} \right) = w(t, x) + \tau_w \frac{\partial w(t, x)}{\partial t}, \quad (2)$$

where $\mu$ is viscosity of fluid, $k$ is coefficient of penetrability, $t$ is time, $\tau_w$ and $\tau_p$ are some positive constants of uniformity of time, $\eta(t)$ is Heavisid function,

$$\eta(t) = \begin{cases} 1, & \text{for } t > 0, \\ 1/2, & \text{for } t = 0, \\ 0, & \text{for } t < 0, \end{cases}$$

$\rho_0$ is density of the fluid in the unperturbed layer conditions, $\beta$ is elastic capacity coefficient of the layer, $\chi = \frac{\kappa}{\mu \beta}$ is coefficient of piezoconductivity of the layer, $\Delta$ is Laplace operator, $p(t, x)$ is pressure, $w(t, x)$ is filtration velocity vector, [1]. Let us consider the initial boundary value problem for the pressure $p(t, x)$ in a bounded domain $\Omega \in R^3$ with a boundary $\partial \Omega$:

$$\chi \Delta \left( p(t, x) + \tau_p \frac{\partial p(t, x)}{\partial t} \right) = \frac{\partial}{\partial t} \left( p(t, x) + \tau_w \frac{\partial p(t, x)}{\partial t} \right),$$
\[ p(0, x) = 0, \]  
\[ a p(t, x) + b \frac{\partial p(t, x)}{\partial n} = \varphi(t, x), \quad 0 < t \leq T, \quad x \in \partial \Omega. \]  
\[ \Delta p^{m+1}(x) - c p^{m+1}(x) = f(p^m(x), p^{m-1}(x), \Delta p^{m-1}(x)), \quad x \in \Omega, m = 1, 2, \ldots, M - 1, \]  
\[ a p^m(x) + b \frac{\partial p^m(x)}{\partial n} = \varphi^m(x), \quad x \in \partial \Omega, \quad m = 0, 1, \ldots, M, \]  
where  
\[ c = \frac{2\tau_w + \tau}{\chi(\tau_p \tau + 2)}, \quad c > 0, \quad \tau_w > 0, \quad \tau_p > 0, \quad \chi > 0, \quad \tau > 0, \quad R = \chi(\tau_w \tau + 2), \]  
\[ f(p^m(x), p^{m-1}(x), \Delta p^{m-1}(x)) = -\frac{4\tau_w}{R} p^m(x) + \frac{2\tau_w - \tau}{R} p^{m-1}(x) + \frac{\chi \tau_p \tau}{R} \Delta p^{m-1}(x). \]  

2 ”Random walk by spheres” algorithm of Monte Carlo methods

The problem (5)–(6) is solved in the same way as the Dirichlet problem for Helmholtz equation, i.e. by the ”random walk by spheres” algorithm [2], [3]. But the boundary is considered as a partially absorbing and partially reflecting. When a ”particle” hits the \( \varepsilon \)-boundary \( \varepsilon \) is the vicinity of the boundary  
\[ \partial \Omega_\varepsilon, \quad \partial \Omega_\varepsilon = \{ x \in \Omega; \quad d(x) < \varepsilon \}, \quad d(x) \]  
the distance from the point \( x \) to the boundary \( \partial \Omega \) of the domain \( \Omega \) of the domain \( \Omega \), a ”particle” is absorbed with the probability  
\[ \frac{|a}{b} |r_i|}{1 + |a| |r_i|} \]  
and reflected to the initial point along the surface normal. Any ”particle” reaches the boundary \( \partial \Omega \) along the surface normal. Here \( r_i \) is the radius of sphere at the