To achieve reliable wind resource assessments, to calculate loads and wakes as well as for precise short-term wind power forecasts, the vertical wind profile above the sea has to be modelled for tip heights up to 160 m. In previous works, we analysed marine wind speed profiles that were measured at the two met masts Horns Rev (62 m high) and FINO1 (103 m) in the North Sea (e.g. [3]). It was shown that the wind shear above 50 m height is significantly higher than expected with Monin-Obukhov corrected logarithmic profiles, revealing an almost linear increase of the wind speed. For that reason, we developed a new analytic model of marine wind velocity profiles, which is based on inertial coupling between the Ekman layers of the atmosphere and the ocean. The good agreement between our theoretical profiles and observations at Horns Rev and FINO1 support the basic assumption in our model that the atmospheric Ekman layer begins at 15 to 30 m height above the sea surface.

5.1 Theory of Inertially Coupled Wind Profiles (ICWP)

The geostrophic wind is regarded as the driving force of the wind field in lower layers of the atmosphere. The momentum is transferred downwards through the Ekman layers of atmosphere and ocean, which are modelled with a constant turbulent viscosity. The challenge is to derive an adequate description of the coupling between the two Ekman layers, where the idea is to introduce a connecting wave boundary layer with a logarithmic wind profile that reaches only up to a maximal height of 30 m.

In order to derive the coupling relations between the three layers the following similarity assumptions are made. First, close to the surface the ratio between the drift velocities of air, $u_{\text{air}}$, and water, $u_{\text{water}}$, as well as the ratio between the friction velocities is given by $u_{\text{water}}/u_{\text{air}} = \frac{u_*}{u_{\text{air}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_{\text{water}}}} \approx \frac{1}{29}$.
where $u_*$ is the friction velocity of the air flow and $w_*$ the one of the water flow while $\rho_{\text{air}}$ and $\rho_{\text{water}}$ are the respective densities. This is equivalent to assuming that the shear stress is constant across the interface between air and water. The constant stress defines the wave boundary layer, extending from a height $z_R$ above to $-z_B$ below the water level. This wave boundary layer is similar to the surface layer onshore where the logarithmic wind profile is valid.

Second, the inertial coupling relation [1] represents the shear stress in the wave boundary layer in the form of a drag law with regard to the inertially weighted fluid speeds at the limits of this layer, where $K_1$ is a specific drag coefficient: $\tau_{\text{wave}} = K_1[\sqrt{\rho_{\text{air}} u(z_B)} - \sqrt{\rho_{\text{water}} u(-z_B)}]^2$.

Third, analogous to the first similarity relation the turbulent viscosities $\nu_{\text{air}}$ and $\nu_{\text{water}}$ of the two Ekman layers are also assumed to be weighted according to the inverse density ratio of the two fluids: $\frac{\nu_{\text{water}}}{\nu_{\text{air}}} = \frac{\rho_{\text{air}}}{\rho_{\text{water}}}$.

These conditions allow the determination of the velocities of the two fluids at certain heights as an implicit function of a given geostrophic wind. The full profiles can be derived in the following way: Due to the constant shear stress the wind profile in the wave boundary layer has a logarithmic shape for neutral thermal stratification. Non-neutral conditions can be modelled with an additional stability function $\Psi_m$ and the Monin–Obukhov (MO) length $L$. Expressed in a coordinate system where the horizontal component of the stress tensor $\tau_h$ is parallel to the $x$-axis the profile can be written as

$$u(z) = \left( u_L + \frac{u_*}{\kappa} \ln \left( \frac{z}{z_R} \right) + \frac{u_*}{\kappa} \Psi_m \left( \frac{z}{L} \right), v_L \right),$$

and is valid for $z_R \leq z \leq z_B$; $\kappa$ is the von Karman constant and $z_R$ is the theoretical height where the momentum transfer from the air to the wave field is centred. The above mentioned drift velocities are $u_{\text{air}} = u(z_R) = u_L$ and $u_{\text{water}} = u(-z_R) \approx \frac{1}{2} u_L$. As a result of the above assumptions and the choice of the coordinate system the drift $(u_L, v_L)$ at the very small height $z_R$ is collinear to the geostrophic wind: $(u_L, v_L) = \frac{1}{2} \mathbf{G}$, where $\mathbf{G} = (u_g, v_g)$. This theoretical, though curious relation has no relevance in-situ, since neither $(u_L, v_L)$ close to the waves nor $\mathbf{G}$ can be measured directly. Note that in the chosen coordinate system $u_g > 0$ and $v_g < 0$.

Concluding from [2], the relation that connects the friction velocity $u_*$ at the water surface to the geostrophic wind is given by

$$|\mathbf{G}| = \frac{\sqrt{r^2 + 1}}{|r + 1|} \frac{u_*}{\sqrt[4]{K_1}}.$$  

The angle of rotation of the surface stress tensor to the left hand side of the geostrophic velocity (in the northern hemisphere) is $\text{atan}(-1/r)$ with $-1 < r < -\infty$, where $r$ still has to be determined.