Some Well-Balanced Shallow Water-Sediment Transport Models

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Summary. This paper is concerned with the numerical approximation of bed-load sediment transport due to water evolution. We introduce an unified formulation for several bed-load models. Some numerical simulations are presented.\textsuperscript{4}

1 Sediment transport model

In order to understand and predict geomorphological evolutions in coastal seas and estuaries a model, which describes the dynamics of the water motion and bed-load sediment transport movement, is needed.

In this paper, the hydrodynamical model is given by shallow water equations, and the morphological model is modeled using a bed evolution equation. Both systems can be written as a coupled system of conservation laws, with non-conservative products and source terms. The model equations are described in Section 1.4.

1.1 Hydrodynamical model: shallow water equations

The system of equations governing a flow of a shallow layer of fluid through a straight channel with a constant rectangular cross-section is given by the well known shallow water model,

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\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} &= 0, \\
\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{h} + \frac{1}{2} g h^2 \right) &= g h \frac{dH}{dx} - g h S_f.
\end{align*}
\]

(1)

In this system, it is supposed that the fluid is homogeneous and inviscid; coordinate \( x \) refers to the axis of the channel, \( t \) is the time; \( h(x,t) \) is the thickness of the fluid layer and \( q(x,t) \) represents the mass-flow, being \( q(x,t) = h(x,t) u(x,t) \) where \( u(x,t) \) is the velocity of the fluid; \( g \) is gravity and \( H(x) \) the depth function measured from a fixed level of reference \( (A_R) \).

The term \( S_f \) models bottom friction, that it is supposed given by a Manning’s law,

\[
S_f = \frac{g \eta^2 u^2}{R_h^{4/3}},
\]

(2)

being \( \eta \) the Manning coefficient. \( R_h \) is the hydraulic ratio, that can be approximated by \( h \).

To study bed-load sediment transport it is necessary to consider a sediment layer of thickness \( z_b \), and a fixed layer (without sediments), with thickness given by \( z_f = -H + A_R \). In this case, system (1) can be rewritten as,

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} &= 0, \\
\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{h} + \frac{1}{2} g h^2 \right) &= -g h \frac{\partial z_b}{\partial x} + g h \frac{dH}{dx} - g h S_f.
\end{align*}
\]

(3)

Fig. 1. Sediment layer over a fixed bed

1.2 Morphological model

The continuity sediment equation models bed-load sediment transport. The temporal variation of sediment layer must be equal to the total variation of the solid transport.

The expression of the conservation law of sediment volume is given by,

\[
\]