Nonconforming Discretization Techniques for Coupled Problems *

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Summary. Multifield problems yield coupled problem formulations for which nonconforming discretizations schemes and problem-adapted solvers can be used to develop efficient numerical algorithms. Of crucial importance are numerically robust transmission operators based on weak continuity conditions. This paper presents the construction of such operators by means of dual discrete Lagrange multipliers for higher order discretizations and for general quadrilateral triangulations of possibly curved interfaces. Various applications are considered, including aero-acoustics, elasto-acoustics, contact and heat transfer.

Keywords: Domain decomposition, non-matching grids, dual Lagrange multipliers, mortar finite elements, nonconforming discretizations

1 Introduction

The approximative solution of multifield problems is characterized by the necessity of being able to combine different model equations, discretizations, spatial and temporal scales, triangulations, and/or spatial dimensions. The main goal of the project C12 “Nonconforming Discretization Techniques for Coupled Problems” is to cope with this necessity by providing general construction principles for nonconforming discretization techniques based on a geometrical decomposition of the computational domain corresponding to the different interacting fields. The two most important parts for achieving this goal are a rigorous mathematical analysis of the underlying weak formulations and the development of efficient numerical algorithms for the solution of the resulting discrete problems.

Being one of the youngest projects within the Collaborative Research Center 404, C12 started in May 2002. At the beginning of the last funding period in

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January 2004, we could already resort to several mathematically well-founded
discretization methods and solution strategies. However, they had been em-
ployed mostly for comparatively simple applications such as stationary and
linear model equations in two space dimensions in combination with low or-
der finite elements and very basic domain and interface geometries. The main
task for the last funding period was to get rid of these shortcomings. We
performed this task by developing and successfully applying nonconforming
coupling schemes for the solution of transient problem settings such as elasto-
dynamics, acoustic wave propagation, elasto-acoustics, heat conduction, and
electro-magnetism, as well as for nonlinear model equations such as nonlinear
structural mechanics including contact problems. Moreover, we could extend
the framework of mortar finite elements using dual Lagrange multipliers from
low order elements and straight interfaces towards higher order elements and
curvilinear interfaces.

In the following report, we can provide a closer look only on some of
the topics considered within this project. In particular, after presenting some
model problems and the concept of dual Lagrange multipliers in Sect. 2, we ad-
dress the extension of biorthogonal bases to higher order elements and general
surface meshes in Sects. 3, 4 and 5. Section 6 is devoted to implementational
issues, while in Sect. 7 we present several application examples dealing with
aero-acoustics, elasto-acoustics, contact and heat transfer. In Sect. 8, we con-
clude by listing all articles which contributed to achieving the goals of project
C12.

2 Variational Setting

We first introduce the model settings which we use for several of our numeri-
cal illustrations. After that, we illustrate why dual Lagrange multipliers are
an important key to efficiently solve problems discretized by mortar finite
elements.

2.1 Model Problems

For the ease of notation and to avoid technicalities, we restrict ourselves to
the case of two non-overlapping open subdomains $\Omega^m$ and $\Omega^n$ sharing a
common interface $\Gamma$, their union giving the global domain $\Omega$, $\overline{\Omega} = \overline{\Omega^m} \cup \overline{\Omega^n}$. By
taking into account the standard modifications at the cross-points or at the
wire-basket of more than two subdomains, the following considerations apply
analogously to decompositions into many subdomains, [4]. For scalar prob-
lems, we focus on Poisson’s equation. In particular, we seek a scalar function
$u$ as the solution of

$$-\Delta u = f \text{ in } \Omega,$$  

with appropriate boundary conditions on $\partial \Omega$. The Lagrange multiplier $\lambda$
is chosen to be the normal flux through the interface $\Gamma$, i.e., $\lambda = -\partial u / \partial n$, 
