Multilevel Numerical Algorithms and Experiments for Contact Dynamics*

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Summary. Nonlinear frictional contact problems are still a challenging task both from the mathematical and engineering point of view. These problems are of crucial importance in various applications. In this report we study dynamical contact problems on mathematical and experimental aspects. In the mathematical part we present a variationally consistent formulation based on mortar techniques with dual Lagrange multipliers for this type of problems. Furthermore, new optimal a priori and a posteriori error estimates were achieved, and numerical results for nearly incompressible materials are given. To solve the resulting nonlinear algebraic problem, we use a primal-dual active set strategy which can also be interpreted as semismooth Newton method. In combination with optimal multigrid methods, the inexact version of this approach can be regarded as a nonlinear multigrid method, and we end up with an efficient iterative solver. In the engineering part experiments to study the properties of the impact between a rotating disc and an elastic strip are presented. Experimental setup and methods are designed to release the disc with prescribed translational and rotational velocities. The impact event is captured by a high-speed digital camera system. Based on image processing, impact quantities, that is, coefficients of normal and tangential restitution, impulse ratio, rotational velocity change, incidence and rebound angles, are measured. A numerical model for interpreting the experimental data is developed, which can also give some insight into effects of strip flexibility. Results are also compared with those from finite element calculations.

Keywords: Contact experiments, frictional contact, mortar discretization, oblique impact, optimal a priori error estimates, primal-dual active set strategies

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1 Introduction

The investigation of nonlinear frictional contact dynamics plays an important role for a wide range of engineering applications. On one hand, this project is mainly concerned with the mathematical analysis and efficient numerical algorithms, and, on the other hand, the design, realization and analysis of experiments for frictional contact problems.

For such problems, nonconforming domain decomposition techniques provide a powerful tool. In this project we work with variationally consistent formulations for this type of problems based on mortar techniques with dual Lagrange multipliers. Dual Lagrange multiplier spaces were introduced in [72], and they are based on a biorthogonal basis resulting in a diagonal mass matrix. Thus we obtain for each node on the slave side a local non-penetration condition and a local friction condition. Therefore, primal-dual can be applied to solve the discrete nonlinear problem.

New optimal a priori error estimates for the discretization error in the $H^1$-norm for the displacements and in the $H^{-1/2}$-norm for the Lagrange multiplier were achieved. Abstract error estimates for variational inequalities can be found, e.g., in [19, 22] and a priori bounds for the discretization error of unilateral contact problems are given, e.g., in [27]. Recently a lot of work has been done to analyze mortar formulations based on standard Lagrange multipliers. A priori error estimates for the displacements in the $H^1$-norm and for the Lagrange multiplier in the $H^{-1/2}$-norm of order $h^{0.75}$ have been established, see, e.g., [8, 9, 15, 51], under $H^2$-regularity assumption. Using additional regularity assumptions on the Lagrange multiplier, order $h$ has been shown, see, e.g., [15, 29]. Although the order $h$ is optimal, the regularity assumptions are quite strong and restrictive. These first a priori results have been considerably improved during the last couple of years, see, e.g., [7, 9, 30, 64]. Most of the theoretical results are obtained for standard Lagrange multipliers. Here, we have considered the case of linear and quadratic mortar finite elements based on dual Lagrange multipliers. Moreover, we show order $h^{0.5+\nu}$ a priori estimates for the displacements and the Lagrange multiplier if the solution is $H^{3/2+\nu}$-regular, $0 < \nu \leq 0.5$. The techniques are based on introducing locally defined truncation operators measuring the nonconformity of the discretization and on Sobolev–Slobodeckij norms.

To solve the nonlinear problem we use a primal-dual active set strategy to find the actual contact zone and slippery/sticky nodes. We will see that this active set strategy can be interpreted as a semismooth Newton method yielding superlinear convergence. For Newton methods in general and their application to contact problems we refer to [3, 13, 31, 66]. For Coulomb friction we present new efficient algorithms for the two-dimensional and three-dimensional case. Here, the superlinear convergence of the active set strategy also applies. Combining the active set approach with an optimal multigrid method as solver for the resulting linear problems and with inexact techniques leads to an efficient iterative solver for the nonlinear problems. Nonlinear material laws and large