Studies of Viscoelasticity with the QCM

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Abstract  The chapter summarizes the standard model of how acoustic multilayers interact with a quartz crystal microbalance (QCM). In a first step, it is shown how the three formulations around (the mathematical description, the description in terms of acoustic reflectivities, and the equivalent circuit) model correspond to each other. Special emphasis is given to the small-load approximation, which states that the shifts of frequency and bandwidth are about equal to the real and the imaginary parts of the stress–speed ratio (the load) at the crystal surface. The (laterally averaged) stress–speed ratio can be computed for many types of samples (including anisotropic and heterogeneous materials). The small-load approximation is therefore of outstanding importance when employing the QCM in complex environments. The second part of the chapter provides the predictions of the standard model for various geometries. This includes the discussion of slip, of the comparison of optical and acoustic thickness, of electrode effects, of the frequency dependence of the viscoelastic parameters, and of the consequences of a finite contact area. Viscoelastic modeling of QCM data has some pitfalls, which are pointed out. A separate section is devoted to the shortcomings of the small-load approximation (which can be very noticeable) and the amendments to the model accounting for these.

Keywords  Acoustic multilayers · Equivalent circuits · Quartz crystal microbalance · Quartz crystal resonator · Viscoelasticity

Abbreviations

A  Area
a  Amplitude of oscillation at the crystal surface
b  Slip length
c  Speed of sound, \( c = (G/\rho)^{1/2} \)
C1  Motional capacitance
C0  Electrical (parallel) capacitance
D  Dissipation, \( D = Q^{-1} \)
d  Thickness
d0  Thickness of the electrode
d1  Thickness of the film
dq  Thickness of the crystal, \( dq = cq/(2f_f) \)
d26  Piezoelectric strain coefficient, \( d_{26} = 3.1 \times 10^{-12} \) m V\(^{-1}\)
e26  Piezoelectric stress coefficient, \( e_{26} = d_{26}G_q = 9.65 \times 10^{-2} \) C m\(^{-2}\)
e  as an index: Electrode
F  Force
Fex  External force
f  as an index: Film (exception: \( f_f \), frequency of the fundamental)
f  Frequency
fr  Resonance frequency (real part)
f0  Resonance frequency in reference state
fr  Resonance complex frequency, \( fr = f_f + i\Gamma \)
f0  Resonance complex frequency in reference state
f1  A parameter close to the resonance frequency of the fundamental
G  Shear modulus, \( G = G' + iG'' \)
Gq  Shear modulus of AT-cut quartz, \( G_q \approx 29.3 \times 10^9 \) Pa
hq  Half of the thickness of the crystal, \( h_q = dq/2 \)
h  Half of the thickness of a layer
iel  Electrical current