Velocity dependence of atomic friction: Rate theory and beyond

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7.1 Introduction

Macroscopic friction between solids is well known to be both of paramount practical importance and of notorious difficulty regarding its theoretical understanding [1,2]. Here, we restrict ourselves to the simpler case of a microscopic contact in the form of a single asperity. Such studies of frictional forces between nanoscale objects are vital both for engineering of micromechanical devices and advancement of our understanding of the laws of nature acting in the nanoworld. While macroscopic friction involves interactions between numerous asperities of the two contacting surfaces, employing an atomic force microscope (AFM) offers a unique opportunity to probe the frictional forces between a single asperity – the tip of an AFM cantilever – and an atomically flat surface. Therefore the research direction of friction force microscopy (FFM) [3] had been initiated only a year after the invention of the AFM in 1986 [4] and became a subject of intensive studies since then (see the reviews [5–7] and references therein).

The laws of nanofriction differ drastically from those of macroscopic friction. In particular, it has been known from the time of Coulomb that the force of friction between two macroscopic bodies in contact is independent of their relative velocity. In contrast, friction force on the nanoscale exhibits a non-trivial velocity dependence, which will be the subject of the present contribution.

Though simpler than macroscopic friction, the adequate interpretation and modeling of microscopic friction experiments still represents a formidable challenge. In particular, direct molecular dynamics simulations are still very far from reaching experimentally realistic conditions [8,9]. The reason is the enormous time scale separation between molecular vibrations and the very rare slip-events of the AFM tip which still cannot be bridged by today’s computer facilities. Hence, non-trivial theoretical modeling steps are indispensable, in particular the concepts of non-linear stochastic processes [10–14]; the above-mentioned time-scale separation will greatly facilitate the calculations within such a model.

The behavior of an atomic force microscope tip in contact with a uniformly moving atomically clean surface is modeled as one-dimensional Brownian mo-
tion in a potential of the tip-surface interaction and of the elastic forces resulting from the deformation of the cantilever, the tip, and the surface in the contact region. A theoretical description of friction force microscopy experiments within such a model is derived on the basis of microscopic considerations. At the focus of this review is the relation between the pulling velocity and the time-averaged lateral force developed in the cantilever, which equals in magnitude the force of friction.

An exact analytical force-velocity relation can be found for asymptotically small cantilever stiffness and high damping. For an arbitrary stiffness, one needs to resort to an approximate treatment. A particularly successful approximation is possible in the stick-slip regime of the tip motion, when the elastic force exhibits a random sawtooth-like time-dependence resulting from the thermally activated transitions of the tip from one surface site to the next. This regime can be treated within the framework of Kramers’ rate theory of thermally activated transitions. The range of validity of such a rate approach in the context of friction force microscopy is discussed. An approximate analytic formula relating the pulling velocity and the average elastic force is derived and its high accuracy is demonstrated numerically. Within the stick-slip regime, the average lateral force increases approximately logarithmically with velocity. While the rate description is applicable when the pulling velocity is not too high, going beyond the stick-slip regime results in a maximum of the average force as a function of velocity, followed by a subsequent decrease. This theoretically predicted non-monotonic force-velocity relation should be observable under realistic experimental conditions.

### 7.2 Experimental Set-Up

In a typical FFM experiment [3], the tip of an AFM is brought in contact with an atomically clean surface moving at a constant velocity \( v \) by means of a normal load \( F_N \) (see Fig. 7.1a). The interaction between the tip and the surface leads to a torsional deformation of the cantilever. One can determine the magnitude of this deformation by optical means and thus deduce the resulting elastic force \( f(t) \), which, by Newton’s third law, equals the instantaneous force of friction. As a rule, the temporal evolution of the friction force proceeds in a sawtooth-like pattern (see Fig. 7.1b showing the results of our numerical simulations; the experimentally observed force evolution is similar, see e.g. [15]). This type of motion of the AFM cantilever is called stick-slip motion. The central quantity of interest is the behavior of the time-averaged friction force

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