0-D Feature in 3D Planar Polygon Testing for 3D Spatial Analysis

Chen Tet Khuan, and Alias Abdul-Rahman

Department of Geoinformatics, Faculty of Geoinformation Science and Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Malaysia {kenchen, alias}@fksg.utm.my

Abstract

In this paper, we discuss the problem of 0D feature-in-3D planar polygon. It has been recognized that 3D union and 3D intersections are among important spatial operators in 3D GIS. A 0D feature-in-polygon test is one of the problems in 3D GIS. Though it is a classic problem, however, it has not been addressed appropriately in the literature. Moreover, the 0D feature-in-polygon query is rather complicated if it is implemented into 3D spatial information system. From the aspect of 3D spatial analysis, the general 0D feature-in-3D planar polygon problem should be formulated in a suitable way. Our basic idea is to solve a general 0D feature-in-3D planar polygon problem that includes all special conditions. The method addresses an essential mathematical algorithm that applicable for real objects and provides an approach for implementation in further 3D analytical operation, e.g. 3D union or 3D intersection for 3D GIS.

KEY WORDS: 0D feature, 3D planar polygon, and 3D GIS

1 Introduction

In GIS, a very natural problem that implements the field of computational geometry is the determination of 0D feature in polygon. By given a 0D feature, $N$ and an arbitrary closed polygon $P$ represented as an array of $n$ points, $P_0, P_1, \ldots, P_{n-1}, P_n = P_0$, determine whether $N$ is inside or outside the polygon $P$. In literature (Foley, et. al, (1990); Haines, (1994b); Harrington, (1983); Nievergelt and Hinrichs, (1993); O'Rourke, (1998);
Sedgewick, (1998); Weiler, (1994); Woo et. al, (1997)), two main definitions can be found. The first one is the even-odd, in which a line is drawn from \( N \) to some other point \( S \) that is guaranteed to lie outside the polygon. If this line \( NS \) crosses the edges \( e_i = P_iP_{i+1} \) of the polygon an odd number of times, the \( N \) is inside \( P \), otherwise it is outside (see Figure 1a). This rule can easily be turned into an algorithm that loops over the edges of \( P \), decides for each edge whether it crosses the line or not, and counts the crossings. We discuss these issues in detail in Section 4. The second one is based on the winding number of \( N \) with respect to \( P \), which is the number of revolutions made around that point while traveling once along \( P \). By definition, \( N \) will be inside the polygon, if the winding number is nonzero, as shown in Figure 1(b).

From these 2 methods, various implementations of these strategies exist (Franklin, (2005); Haines, (1994a); Haines, (1994b); Mehlhorn and Näher, (1999); O'Rourke, (1998); Sedgewick, (1998); Stein, (1997); Theoharis and Böhm, (1999)), which differ in the way to compute the intersection between the line and an edge in order to determine whether the 0D feature is inside or outside the polygon. However, some of the problems need to be investigated, since these determination strategies are rather fragmented, i.e. in term of applying into 3D planar polygon instead of 2D, or if \( y \)-intercept crosses the vertices or line of 3D polygon in certain special cases.

![Fig. 1. Determination of 0D feature in polygon, (a) odd-even, and (b) winding methods](image-url)

In this paper, we concentrated on simple but complete strategy in determining the 0D feature inside or outside 3D planar polygon. The algorithm will fully cover the third dimension in order to apply in 3D situation.

The paper is organized in the following order: first, short discussion of the 3D plane equation is given in section 2. This discussion will cover some examples in order to verify the mathematical function. Then, the de-