A Sample cohomology calculations

The cohomology rings of over a hundred small $p$-groups may be found on the World Wide Web at the address

http://www.math.uni-wuppertal.de/~green/Coho/index.html

Some salient samples are printed in this appendix.

A.1 The cyclic group of order 2

$G$ is $C_2$, the cyclic group of order 2.
This cohomology ring is well known. It has one generator: $y$ in degree 1. There are no relations.

A.2 The cyclic group of order 4

$G$ is $C_4$, the cyclic group of order 4.
This cohomology ring is well known. It has 2 generators:
1. $y$ in degree 1
2. $x$ in degree 2
There is one minimal relation: $y^2 = 0$.
This minimal resolution constitutes a Gröbner basis for the ideal of relations.

A.3 The Klein 4-group

$G$ is $V_4$, the elementary abelian group of order 4.
This cohomology ring is well known. It has 2 generators:
1. $y_1$ in degree 1
2. $y_2$ in degree 1
There are no relations.
A.4 The dihedral group of order 8

$G$ is $D_8$, the dihedral group of order 8. It is the third Small Group of order 8 and its Hall–Senior number is 4. $G$ has rank 2, 2-rank 2 and exponent 4. Its centre has 2-rank 1. The 3 maximal subgroups are: $C_4, V_4 \times (2 \times )$. There are 2 conjugacy classes of maximal elementary abelian subgroups. Every such subgroup has 2-rank 2. This cohomology ring is well-known, and was successfully calculated.

Ring structure

The cohomology ring has 3 generators:

1. $y_1$ in degree 1
2. $y_2$ in degree 1
3. $x$ in degree 2, a regular element

There is one minimal relation: $y_1 y_2 = 0$. This minimal relation constitutes a Gröbner basis for the ideal of relations.

*Essential ideal* Zero ideal.

*Nilradical* Zero ideal.

Completion information

For this cohomology computation the minimal resolution was constructed out to degree 4. The presentation of the cohomology ring reaches its final form in degree 2. Carlson’s criterion detects in degree 4 that the presentation is complete. This cohomology ring has dimension 2 and depth 2. A homogeneous system of parameters is

$h_1 = x$ in degree 2

$h_2 = y_2^2 + y_1^2$ in degree 2

The first two terms $h_1, h_2$ constitute a regular sequence of maximal length. The first term $h_1$ constitutes a complete Duflot-regular sequence. That is to say, its restriction to the greatest central elementary abelian subgroup constitutes a regular sequence of maximal length.

*Essential ideal* The essential ideal is the zero ideal.