Evaluating Kos in a Neutral Threat Environment: Preliminary Results

William L. Spight

Oakland, California
BillSpight@aol.com

Abstract. The idea of a komaster made it possible to use thermography to analyze loopy games, called kos, in Go. But neither player may be komaster. In that case a neutral threat environment permits the construction of thermographs for kos. Using neutral threat environments some hyperactive kos are evaluated and general results are derived for approach kos.

1 Introduction

1.1 Combinatorial Games

Combinatorial games are 2-person games of perfect information [1]. The players, called Left and Right (Black and White, respectively, in Go), alternate play until one player has no move, and therefore loses. Numbers are games, and the play may end when a number, or score, is reached. Scores in favor of Left are positive. Independent games may be combined in a sum. Play alternates in the sum, but not necessarily in each component of the sum. Thus, which player has the move is not part of the definition of a combinatorial game.

Equivalent games need not have the same form, but each game has a simplest, or canonical, form, and equivalent games have the same canonical form. The canonical form for 0, for instance, is \{\}. The vertical slash indicates the root of the game tree. Options for Left are to the left of the slash, and Right’s options to the right. Here, neither player has a move. Zero is a win for the second player.

A parent node has more slashes than its children, or followers. In the game \(G = \{3|1||− 4\}\), Right can play to a score of -4 and Left can play to the game \(H = \{3|1\}\). To find the negative of a game, flip it from left to right and negate any scores or other subgames. The negative of \(G\) is \(\{4|| − 1| − 3\}\).

1.2 Mean Value and Temperature

What does Left gain from a play from \(G\) to \(H\)? Strictly speaking, it is their difference, \(H − G\). But we may also speak of the average gain in terms of the mean values of \(G\) and \(H\). The mean value of \(G\) is not apparent, but \(H = \{3|1\}\) has an obvious mean value of 2. Each player gains 1 point on average by a play in \(H\), and this gain indicates the urgency of a play in \(H\). This urgency is called
the *temperature* of $H$. The mean value of a game is not always the average of the mean values of two of its followers, nor is its temperature always the difference between it and the mean value of an immediate follower.

### 1.3 Thermography

Thermography is a method for finding the mean value and temperature of a combinatorial game [1]. The thermograph of a game, $G$, represents the results, $v$, of optimal play (which may be no play at all) in that game for each temperature $t$. By convention $t$ is plotted on the vertical axis with positive values above the origin, and $v$ is plotted on the horizontal axis with positive values to the left of the origin. $t$ may be considered a tax on each play. The imposition of the tax is called *cooling*. For each $t$, the left wall of the thermograph, $LW(G)$, indicates the result with optimal play if Left (Black) plays first, the right wall, $RW(G)$, the result with optimal play if Right (White) plays first. If the tax is high enough, the player will choose not to make a play. At the top of each thermograph neither player can afford to play, and the walls coincide in a vertical mast, $v = m(G)$. $m(G)$ is called the mast value of the game. (For classical combinatorial games the mean value and mast value are the same, but that may not be the case for some Go positions involving loopy games called *kos*.) The temperature at the bottom of this vertical mast is called the temperature of the game, $t(G)$.

![Thermograph of \{3\|1\| - 4\}](image)

Figure 1 shows the thermograph of $G$. $RW(G) = -4 + t$, when $0 \leq t \leq 3$. When $0 \leq t \leq 1$, $LW(G)$ is vertical with $v = 1$, indicating that Right will reply if Left plays to $\{3\|1\}$, and is the line $v = 2 - t$ when $1 \leq t \leq 3$. The two walls form a mast at temperature 3. $t(G) = 3$ and $m(G) = -1$.

Note that below $t(G)$, $LW(G) = RW(H) - t$; similarly, $RW(G) = LW(-4) + t$ below $t(G)$. (The left wall of a number is simply its mast.) The thermograph of a non-ko game is derived directly from the thermographs of its followers. The left scaffold of $G$, $LS(G)$, is the maximum of the right walls of the left followers of $G$ minus $t$. Likewise, $RS(G)$ is the minimum of the left walls of the right followers of $G$ plus $t$. The intersection or coincidence of the scaffolds determines $t(G)$ and $m(G)$. 