3 Overview of Freeway Traffic Theories and Models: Fundamental Diagram Approach

3.1 Introduction:
Hypothesis About Theoretical Fundamental Diagram

A huge number of models have been developed to explain empirical spatiotemporal traffic dynamics (e.g., [1–11,13–18,20,21,42–46,362–367,370–395, 397–401,420–422,436–465]; see also references in the reviews by Gartner et al. (eds.) [31], Wolf [32], Chowdhury et al. [33], Helbing [34,35], Nagatani [36], Nagel et al. [38]).

In 1955 Lighthill and Whitham [2] wrote in their classic work (see p. 319 in [2]): “…The fundamental hypothesis of the theory is that at any point of the road the flow (vehicles per hour) is a function of the concentration (vehicles per mile)…” (Fig. 3.1).

To explain this fundamental hypothesis, let us consider hypothetical model solutions where all vehicles move at the same distances with respect to one another and with the same time-independent vehicle speed. These solutions are called “homogeneous,” “equilibrium,” “stationary” states, or

![Fig. 3.1. Qualitative example of the theoretical fundamental diagram [2]. A steady model state where the vehicle speed $v$ is related to only one vehicle density, which is given by the intersection of the dotted line “slope $v$” with the flow–density relation, i.e., with the fundamental diagram](image-url)
“steady-state” model solutions. In this book, we call these hypothetical traffic states “steady states.” The fundamental hypothesis about the fundamental diagram means that all steady-state model solutions lie on a theoretical fundamental diagram, i.e., on a curve(s) in the flow–density plane (Fig. 3.1) [31, 33, 35, 36, 38]. This approach to traffic flow modeling and theory whose fundamental hypothesis is associated with the fundamental diagram for steady-state model solutions can be called the fundamental diagram approach. Thus, in this approach [31, 33, 35, 36, 38] the whole multitude of the steady-state model solutions cover a one-dimensional region in the flow–density plane.

Concerning the hypothesis about the theoretical fundamental diagram, it must be noted that complex spatiotemporal traffic patterns are observed in congested traffic (e.g., [80, 82, 85–88, 218]). Thus, at higher density the empirical fundamental diagram is related to averaged characteristics of spatiotemporal congested patterns measured at a freeway location rather than to features of the hypothetical steady states of congested traffic. This means that the existence of the theoretical fundamental diagram is only a hypothesis. The hypothesis about the theoretical fundamental diagram underlies almost all traffic flow modeling approaches up to now [31, 33, 35, 36] in the sense that the models are constructed such that in the unperturbed, noiseless limit they have a fundamental diagram of steady states, i.e., the steady states form a curve in the flow–density plane (e.g., [2–4, 6–11, 13–18, 20, 21, 42–46, 362–367, 370–395, 397–401, 420–422, 436–464]; see also references in the reviews [31–36, 38]).

In this chapter, the following main points are considered:

(i) achievements of the fundamental diagram approach, which is the theoretical basis for most of the earlier mathematical traffic flow models and theories;

(ii) a critical analysis of the fundamental diagram approach for a mathematical description of spatiotemporal features of congested traffic.

### 3.2 Achievements of Fundamental Diagram Approach to Traffic Flow Modeling and Theory

The first models of traffic flow in the fundamental diagram approach were based on collective properties of traffic such as conservation of the number of vehicles, the balance of average vehicle speed, and other more complex macroscopic properties of the flow. The first macroscopic models were proposed in 1955 by Lighthill and Whitham [2], in 1956 by Richards [3], by Prigogine in 1959 [14], and by Payne in 1971 [15, 16]. The Payne model was further developed by many authors (e.g., [27, 363, 366, 367, 395]).

There has also been a huge number of microscopic models in which the individual behavior of each vehicle is taken into account. Examples include the