5 Soft Computing Based Decision Making and DSS

5.1 Fuzzy Linear Programming

Statement of fuzzy linear programming

A classical (crisp) linear programming problem can be written as

\[
\begin{align*}
\text{maximize} & \quad Z = cx \\
\text{subject to} & \quad Ax \leq b, \quad x \geq 0,
\end{align*}
\]

(5.1)

where \( Z \) denotes the objective, \( Ax \leq b \) represents the set of (rigid) constraints and \( x \) are the structural variables.

In order to understand where and how fuzziness is applied in the linear programming problem, let us consider a simple example.

Manufacturing company must determine the mix of its commercial products \( B \) and \( C \) to be produced next year. The company produces two product lines, the \( B \) and the \( C \). The average profit is $400 for each \( B \) and $800 for each \( C \).

Fabrication and assembly are limited resources. There is a maximum of 5,000 hours of fabrication capacity available per month (each \( B \) requires 3 hours and each \( C \) requires 5 hours). There is a maximum of 3,000 hours of assembly capacity available per month (each \( B \) requires 1 hour and each \( C \) requires 4 hours).

How many of each products should be produced each month in order to maximize profit?

The objective function is

\[
\text{maximize} \quad Z = 400x_1 + 800x_2
\]

where \( Z \) = the monthly profit from \( B \) and \( C \); \( x_1 \) = the number of \( B \) produced each month; \( x_2 \) = the number of \( C \) produced each month. We have following constraints

<table>
<thead>
<tr>
<th>B(( x_1 ))</th>
<th>C(( x_2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required Time/Unit</td>
<td>Required Time/Unit</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

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The company manager formulates the production scheduling problem as follows:

\begin{align}
3x_1 + 5x_2 & \leq 5,000 & \text{Fab} \\
x_1 + 4x_2 & \leq 3,000 & \text{Assy}
\end{align} \tag{5.2}

\[ x_1, x_2 \geq 0 \quad \text{Nonnegativity} \]

However, the total available labor hours may not be that precise, because the company manager can ask workers to work overtime. Therefore, some tolerance would be put on the constraints. So, the of fuzziness appears in the specification of resources.

Since the market changes constantly, it may not be certain that the per unit profit of B and C are 400 and 800, respectively. The numbers could only be viewed as the most possible values. Therefore, it is reasonable to represent the objective coefficients as fuzzy numbers.

Because of the inconsistence of human workers, value of labor hours in making B and C could only be viewed as around 3 and 5 of materials. Therefore, it is reasonable to represent these coefficients by fuzzy numbers.

If we assume that the Linear Programming (LP) decision has to be made in fuzzy environment, then there exist several modification of (5.1). Now different models have been suggested which allow fuzziness in the objective and/or constraints in traditional LP. Some of those are considered below.

1. LP problem with fuzzy goal:
   \[
   \text{maximize } Z = \tilde{c}x \\
   \text{s.t. } Ax \leq b, \\
   x \geq 0.
   \]

2. LP problem with fuzzy resources:
   \[
   \text{maximize } Z = cx \\
   \text{s.t. } Ax \leq b, \\
   x \geq 0.
   \]

3. LP problem with fuzzy constraint coefficients:
   \[
   \text{maximize } Z = cx \\
   \text{s.t. } \tilde{A}x \leq b, \\
   x \geq 0.
   \]