Modeling and Valuation of Credit Risk

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1 Introduction

The goal of this work is to present a survey of recent developments in the area of mathematical modeling of credit risk and credit derivatives. Credit risk embedded in a financial transaction is the risk that at least one of the parties involved in the transaction will suffer a financial loss due to decline in the creditworthiness of the counter-party to the transaction, or perhaps of some third party. For example:

- A holder of a corporate bond bears a risk that the (market) value of the bond will decline due to decline in credit rating of the issuer.
- A bank may suffer a loss if a bank’s debtor defaults on payment of the interest due and (or) the principal amount of the loan.
- A party involved in a trade of a credit derivative, such as a credit default swap (CDS), may suffer a loss if a reference credit event occurs.
- The market value of individual tranches constituting a collateralized debt obligation (CDO) may decline as a result of changes in the correlation between the default times of the underlying defaultable securities (i.e., of the collateral).

The most extensively studied form of credit risk is the default risk – that is, the risk that a counterparty in a financial contract will not fulfil a contractual commitment to meet her/his obligations stated in the contract. For

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this reason, the main tool in the area of credit risk modeling is a judicious specification of the random time of default. A large part of the present text will be devoted to this issue, examined from different perspectives by various authors.

Our main goal is to present the most important mathematical tools that are used for the arbitrage valuation of defaultable claims, which are also known under the name of credit derivatives. We decided to examine the important issue of hedging credit risk in a separate work (see the forthcoming paper by Bielecki et al. (2004)).

These lecture notes are organized as follows. First, in Chapter 1, we provide a concise summary of the main developments within the so-called structural approach to modeling and valuation of credit risk. This was historically the first approach used in this area, and it goes back to the fundamental papers by Black and Scholes (1973) and Merton (1974). Since the main object to be modeled in the structural approach is the process representing the total value of the firm’s assets (for instance, the issuer of a corporate bond), this methodology is frequently termed the value-of-the-firm approach in financial literature.

Chapter 2 is devoted to the intensity-based approach, which is also known as the reduced-form approach. This approach is purely probabilistic in nature and, technically speaking, it has a lot in common with the reliability theory. Since, typically, the value of the firm is not modeled, the specification of the default time is directly related to the likelihood of default event conditional on an information flow. More specifically, the default risk is reflected either by a deterministic default intensity function, or, more generally, by a stochastic intensity.

The final chapter provides an introduction to the area of modeling dependent credit migrations and defaults. Arguably, this is the most important and the most difficult research area with regard to credit risk and credit derivatives. We describe the case of conditionally independent default time, the copula-based approach, as well as the Jarrow and Yu (2001) approach to the modeling of dependent stochastic intensities. We conclude by summarizing one of the approaches that were recently developed for the purpose of modeling term structure of corporate interest rates.

Acknowledgments

Since this is a survey article, we do not provide here, with rare exceptions, the proofs of mathematical results that are presented in the text. For the demonstrations, the interested reader is referred to numerous original papers, as well as recent monographs, which are collected in the (non-exhaustive) list of references. Let us only mention, that the proofs of most results can be found in Bielecki and Rutkowski (2002) and Jeanblanc and Rutkowski (2000, 2001).

Finally, it should be acknowledged that some results (especially within the intensity-based approach presented in Chapter 2) were obtained independently by various authors, who worked under different sets of assumptions.