Projective Reconstruction of Surfaces of Revolution

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Abstract. This paper addresses the problem of recovering the generating curve of a surface of revolution from a single uncalibrated perspective view, based solely on the object’s outline and two (partly) visible cross-sections. Without calibration of the camera’s internal parameters such recovery is only possible up to a particular transformation of the true shape. This is however sufficient for 3D reconstruction up to a 2 DOF transformation, for recognition of objects, and for transfer between views. We will describe the basic algorithm and show some examples.

1 Introduction

This paper describes a method for the recovery of the generating function of a Surface of Revolution (SOR) from a single image. Ignoring the usual unknowns scale and position the generating function could be used for a reconstruction up to a particular subgroup of 3D-projective transformations with only 2 degrees of freedom (DOF). Additionally, the generating function can be used directly for recognition of the SOR from an arbitrary view point, provided the method of matching plane curves is invariant to a projective transformation. Transfer of the contour between views finally can be used for verification.

Most algorithms for the reconstruction of SORs (or, more generally, Straight Homogeneous Generalised Cylinders, SHGCs) go back to \([1]\), where a possible algorithm for the identification of a SHGC’s axis from its outline was given, on to an algorithm for the identification of a SHGC’s ending cross-sections in \([2]\), and \([3,4,5,6]\) who all gave algorithms for the reconstruction of SHGCs from orthographic views (e.g. approximated by a tele-lens). For SORs the approach in \([5,6]\) was recently extended to work with any calibrated camera \([7]\). All of these algorithms require knowledge about the actual camera used, and mostly a particular camera geometry, in addition to the object’s outline and at least one (partly) visible cross-section. The cross-section could be a discontinuity in the SOR’s generating function such as the flat top or bottom of an object, or surface markings. Our novel algorithm by contrast requires the outline and two partly visible cross-sections, but no additional information about the imaging process.
Fig. 1. SORs as either a rotational surface or as a stack of circular cross-sections.

— it is in fact invariant not only to perspective transformation, but the entire projective group.

Other methods for the reconstruction of SORs previously employed by us were e. g. based on a number of distinguished points [8,9]. In [10] we gave an algorithm for the computation of an SOR’s projectively (quasi-) invariant signature and in [11] these methods demonstrated a recognition-rate similar to that of appearance-based approaches. Since we already demonstrated in [12,6] that the identification and grouping of potential SORs in cluttered images is possible, we will in this paper concentrate on simple images so as not to confound the issues.

This paper is structured as follows: we will recall the basic properties of an SOR in Sec. 2. Section 3 gives a description of the actual algorithm used for the projective reconstruction as well as some examples, and applications to recognition and transfer will be discussed in Sec. 4.

2 Object Model

There are two traditional models for the construction of Surfaces of Revolution. Most commonly used is that of a generating function $r(z)$ being rotated around the axis of revolution, resulting in a surface

$$S = (r(z) \cos(\phi), r(z) \sin(\phi), z)^T,$$

compare Fig. 1. Our intentions, however, are best served if we understand an SOR as a special case of a Straight Homogeneous Generalised Cylinder, namely one with a circular cross-section where the axis goes through the centre of the cross-section, and the cross-section is orthogonal to the axis. In such a model the circular cross-section is swept along the SOR’s axis and scaled according to a scaling function (equivalent to the planar generating function mentioned above); further simplifying this model a little, we can imagine an SOR to be constructed out of (infinitely) many circular cross-sections stacked on top of each other. Figure 1 illustrates both models. The idea of forming the SOR outline as an envelope of circles is the backbone of the novel method presented here.