An Efficient Solution to the Corridor Search Problem

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Abstract. The corridor search problem is the problem of searching for a mobile intruder in the corridor, which is a polygonal region $P$ with an entrance $u$ and an exit $v$, by the mobile searcher having flashlights whose visibility is limited to the rays emanating from his position. In this paper, we relate the corridor search problem to the well studied two-guard problem, which gives us efficient solutions for several versions of the corridor search problem. Specially, we can decide whether there exists a schedule for the searcher having two flashlights to detect the intruder in $O(n \log n)$ time, and if so generate such a schedule in $O(n \log n + k)$ time, where $k (\leq n^2)$ is the minimum number of search instructions. Our results improve upon the previous time bounds $O(n^2)$ and $O(n^2 \log n)$, respectively.

1 Introduction

In recent years, much attention has been devoted to the problem of searching for a mobile intruder in a polygonal region $P$ by the mobile searcher having flashlights whose visibility is limited to the rays emanating from his position [2, 5, 9, 10]. The goal is to decide whether there exists a schedule for the searcher to detect the intruder, no matter how fast he moves, and if so generate such a schedule. This problem, called the polygon searching problem, was introduced by Suzuki and Yamashita [9]. Both the searcher and the intruder are modeled by points that can move continuously in $P$. A searcher is called the $k$-searcher if he has $k$ flashlights and can see along $k$ rays emanating from his searchlights, or the $\infty$-searcher if he has a light bulb and is of a 360° field of vision. A flashlight can be rotated continuously with bounded speed to change its direction. A polygon is said searchable by a given searcher if there exists a schedule for the searcher. A number of necessary conditions and sufficient conditions for a polygon to be searchable by a $k$-searcher or an $\infty$-searcher are given in [9].

Crass, Suzuki and Yamashita considered a variant of the polygon search problem, called the corridor search problem, in which the given polygon $P$ has an entrance $u$ and an exit $v$ [2]. The task of the searcher, starting at $u$, is then to force the intruder, out of $P$ through $v$ (but not $u$). They show that the 2-searcher has the same capability as the $\infty$-searcher in the corridor search problem, and give a necessary and sufficient condition for a corridor to be searchable by a 2-searcher. Also, an $O(n^2)$ time algorithm for determining whether the given corridor is searchable by a 2-searcher and an $O(n^2 \log n)$ time algorithm for generating a search schedule are presented.

In this paper, we relate the corridor search problem to the well studied two-guard problem [7, 6], which gives us efficient solutions for several versions of the
corridor search problem. Specially, we can decide whether there exists a schedule for the 2-searcher to detect the intruder in $O(n \log n)$ time, and if so generate such a schedule in $O(n \log n + k)$ time, where $k (\leq n^2)$ is the minimum number of search instructions. Our results improve upon the previous time bounds $O(n^2)$ and $O(n^2 \log n)$, respectively.

2 Basic definitions

Let $P$ denote a simple polygon in the plane, i.e., a polygon without self-intersections or holes. Two points $x, y \in P$ are said to be mutually visible if the line segment $\overline{xy}$ connecting them is entirely contained within $P$. For two regions $R, Q \subseteq P$, we say that $R$ is weakly visible from $Q$ if every point in $R$ is visible from some point in $Q$.

When two vertices $u$ and $v$ of polygon $P$ are given, the boundary of $P$ is divided into two polygonal chains, $L$ and $R$, with common endpoints $u$ and $v$. Both chains $L$ and $R$ are oriented from $u$ to $v$. Points on $L$ ($R$) are denoted by $p_1, p_2, \ldots, p_{|L|}$ ($q_1, q_2, \ldots, q_{|R|}$). For a vertex $x$ of a polygonal chain, $Succ(x)$ denotes the vertex of the chain immediately succeeding $x$, and $Pred(x)$ the vertex immediately preceding $x$. For two points $p, p' \in L$, we say that $p$ precedes $p'$ (and $p'$ succeeds $p$) if we encounter $p$ before $p'$ when traversing $L$ from $s$ to $t$. We write $p < p'$. The chain $L_{<p}$ ($L_{>p}$) is the subchain of $L$ consisting of all points that precede (succeed) $p$. The definition for $R$ is symmetric.

A vertex of $P$ is reflex if its interior angle is greater than $180^\circ$; otherwise, it is convex. An important definition for reflex vertices is that of ray shots: the backward ray shot from a reflex vertex $r$ of chain $L$ or $R$, denoted by $Backw(r)$, is the first point of $P$ hit by a “bullet” shot at $r$ in the direction from $Succ(r)$ to $r$, and the forward ray shot $Forw(r)$ is the first point hit by the bullet shot at $r$ in the direction from $Pred(r)$ to $r$.

Let $\partial P$ denote the boundary of polygon $P$. A search schedule of the $k$-searcher for $P$ from $u$ to $v$ is a tuple $S = (s, f_1, f_2, \ldots, f_k)$ of $k+1$ continuous functions $s : [0, 1] \to P$ and $f_1, f_2, \ldots, f_k : [0, 1] \to \partial P$, where $s(0) = f_1(0) = \cdots = f_k(0) = u$ and $s(1) = f_1(1) = \cdots = f_k = v$. (We have assumed in the definition that any ray emanating from a flashlight hits the boundary of $P$ instantly.) A point $x \in P$ is illuminated at time $t$ during the execution of $S$ if $x$ lies on one of the line segments $s(t)f_1(t), s(t)f_2(t), \ldots, s(t)f_k(t)$, where $s(t)$ is the position of the searcher and $f_1(t), f_2(t), \ldots, f_k(t)$ are the endpoints of flashlight on the boundary of $P$ at time $t$, respectively. Hence, the illuminated points at any given time are those visible from the $k$-searcher. Any region that might contain the intruder at a time is said to be contaminated; otherwise it is said to be clear. Obviously, the line segments connecting the searcher and his flashlight at any time of a search schedule should separate the clear regions from the contaminated ones. A schedule of the $\infty$-searcher can be given analogously [9]. Corridor $P$ is said to be $k$-searchable (or $\infty$-searchable) if there exists a search schedule of the $k$-searcher (or $\infty$-searcher) for $P$. 