7 Dynamic Characteristics of Mechanisms with Rigid Links

For an assessment of the quality of a mechanism, it is necessary to proceed from the acting forces and from the constraint reactions obtained in the force analysis, to some general dynamic criteria which reflect the most important properties of a mechanism in the most typical dynamic regimes. In this chapter we will consider methods of defining dynamics characteristics as well as methods of improving the qualities of a mechanism through a modification of its parameters and through the introduction of certain complementary devices.

7.1 Internal Vibration Activity of a Mechanism

Let us consider a cyclic mechanism with rigid links and with ideal kinematic pairs (Fig. 7.1), which is a combination of a transmission mechanism with transmission ratio \( i \) (\( \varphi = q / i \)) with an actuating mechanism \( \varphi \) with a nonlinear position function.

Assuming that the generalized resistance force can be represented in the form (6.12), we write the equation of motion of the mechanism in the form (6.6):

\[
J(q) \ddot{q} + \frac{1}{2} J'(q) \dot{q}^2 = Q + Q_R(q, \dot{q}).
\]

(7.1)

The reduced moment of inertia \( J(q) \) can be represented in the form (6.8); in what follows, this series expansion is abbreviated.
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\[ J(q) = J_0 + \tilde{J}(q) = \frac{1}{2\pi i} \int_0^{2\pi i} J(q) \, dq + \tilde{J}(q), \quad (7.2) \]

where the variable part of the reduced moment of inertia of the mechanism is \( \tilde{J}(q) \) which, in the present case, has period \( 2\pi i \). The reduced moment of the resistance forces can be represented analogously, also being a periodic function of \( q \) with period \( 2\pi i \):

\[ Q_R(q, \dot{q}) = Q_{R0}(\dot{q}) + \tilde{Q}_R(q, \dot{q}), \quad (7.3) \]

where the leading term

\[ Q_{R0}(\dot{q}) = \frac{1}{2\pi i} \int_0^{2\pi i} Q_{R0}(q, \dot{q}) \, dq \quad (7.4) \]

is the average value of the reduced moment of resistance forces and \( \tilde{Q}_R = Q_R(q, \dot{q}) - Q_{R0}(\dot{q}) \).

One of the most typical performances of a cyclic mechanism is the steady-state motion when the angular velocity of the input link is close to some constant value \( \dot{q} = \omega_0 \). Let us consider a mechanism characteristic which reflects the dynamic properties of the mechanism during steady-state motion. We assume that the input link rotates with constant angular velocity \( \omega_0 \). Let us find the generalized driving force (moment) to be applied to the input link in order to achieve such a motion. Substituting \( \dot{q} = \omega_0, \, q = \omega_0 t, \, \ddot{q} = 0 \) into (7.1), we have

\[ Q(t) = \frac{1}{2} J'(\omega_0 t)\omega_0^2 - Q_R(\omega_0 t, \omega_0) = -Q_{R0}(\omega_0) + \tilde{Q}(t), \quad (7.5) \]

where \( \tilde{Q}(t) \) is the variable part of the driving moment. The opposite moment

\[ L(t) = -\tilde{Q}(t), \quad (7.6) \]

acted upon the engine by the mechanical system, is called perturbation moment. The ability of the mechanism to generate a variable perturbation moment during a uniform rotation of the input link reflects its internal vibration activity. From the expression (7.5) it is seen that the internal vibration activity of the system is caused by the variability of the reduced moment of inertia of the mechanism and by the explicit dependence of the reduced moment of resistance forces on the coordinate \( q \).

The perturbation moment is a periodic function of \( t \) with period \( T = 2\pi i / \omega_0 = 2\pi / \nu \), where \( \nu \) is the angular velocity of the input link of the actuating mechanism. This moment does not have a constant term. It can be represented in the form of a Fourier series: