The fundamental principles of charged particle beam dynamics as discussed in previous chapters can be applied to almost every beam transport need. Focusing and bending devices for charged particles are based on magnetic or electric fields which are specified and designed in such a way as to allow the application of fundamental principles of beam optics leading to predictable results.

Beam transport systems can be categorized into two classes: The first group is that of beam transport lines which are designed to guide charged particle beams from point $A$ to point $B$. In the second class, we find beam transport systems or magnet lattices forming circular accelerators. The physics of beam optics is the same in both cases but in the design of actual solutions different boundary conditions may apply. Basic linear building blocks in a beam transport line are the beam deflecting bending magnets, quadrupoles to focus the particle beam, and field free drift spaces between magnets. Transformation matrices have been derived in Chap. 4 and we will apply these results to compose more complicated beam transport systems. The arrangement of magnets along the desired beam path is called the magnet lattice or short the lattice.

Beam transport lines can consist of an irregular array of magnets or a repetitive sequence of a group of magnets. Such a repetitive magnet sequence is called a periodic magnet lattice, or short periodic lattice and if the magnet arrangement within one period is symmetric this lattice is called a symmetric magnet lattice, or short a symmetric lattice. By definition a circular accelerator lattice is a periodic lattice with the circumference being the period length. To simplify the design and theoretical understanding of beam dynamics it is customary, however, to segment the full circumference of a circular accelerator into sectors which are repeated a number of times to form the complete ring. Such sectors are called superperiods and define usually most salient features of the accelerator in contrast to much smaller periodic segments called cells, which include only a few magnets.
In this chapter, we concentrate on the study of periodic focusing structures. For long beam transport lines and specifically for circular accelerators it is prudent to consider focusing structures that repeat periodically. In this case, one can apply beam dynamics properties of one periodic lattice structure as many times as necessary with known characteristics. In circular particle accelerators such periodic focusing structures not only simplify the determination of beam optics properties in a single turn but we will also be able to predict the stability criteria for particles orbiting an indefinite number of revolutions around the ring.

To achieve focusing in both planes, we will have to use both focusing and defocusing quadrupoles in a periodic sequence such that we can repeat a lattice period any number of times to form an arbitrary long beam line which provides the desired focusing in both planes.

7.1 FODO Lattice

The most simple periodic lattice would be a sequence of equidistant focusing quadrupoles of equal strength. This arrangement is unrealistic with magnetic quadrupole fields which do not focus in both the horizontal and vertical plane in the same magnet. The most simple and realistic compromise is therefore a periodic lattice like the symmetric quadrupole triplet which was discussed in Sect. 4.2.3. and is shown schematically in Fig. 7.1.

![FODO-lattice](image)

**Fig. 7.1.** FODO-lattice (QF: focusing quadrupole; QD: defocusing quadrupole)

Each half of such a lattice period is composed of a focusing (F) and a defocusing (D) quadrupole with a drift space (O) in between. Combining such a sequence with its mirror image as shown in Fig. 7.1 results in a periodic lattice which is called a FODO lattice or a FODO channel. By starting the period in the middle of a quadrupole and continuing to the middle of the next quadrupole of the same sign not only a periodic lattice but also a symmetric lattice is defined. Such an elementary unit of focusing is called a lattice unit or in this case a FODO cell. The FODO lattice is the most widely used