24.1 Introduction

There are several tasks in the investment management process. These include setting the investment objectives, establishing an investment policy, selecting a portfolio strategy, asset allocation, and measuring and evaluating performance. Bayesian methods have been either used or proposed as a tool for improving the implementation of several of these tasks. There are principal reasons for using Bayesian methods in the investment management process. First, they allow the investor to account for the uncertainty about the parameters of the return-generating process and the distributions of returns for asset classes and to incorporate prior beliefs in the decision-making process. Second, they address a deficiency of the standard statistical measures in conveying the economic significance of the information contained in the observed sample of data. Finally, they provide an analytically and computationally manageable framework in models where a large number of variables and parameters makes classical formulations a formidable challenge.

The goal of this chapter is to survey selected Bayesian applications to investment management. In Section 24.2, we discuss the single-period portfolio problem, emphasizing how Bayesian methods improve the estimation of the moments of returns, primarily the mean. In Section 24.3, we describe the mechanism for incorporating asset-pricing models into the investment decision-making process. Tests of mean-variance efficiency are surveyed in Section 24.4. We explore the implications of predictability for investment management in Section 24.5 and then provide concluding remarks in Section 24.6.

24.2 The Single-Period Portfolio Problem

The portfolio choice problem represents a primary example of decision-making under uncertainty. Let \( r_{T+1} \) denote the vector \((N \times 1)\) of next-period returns and \( W \)
current wealth. We denote next-period wealth by $W_{T+1} = W(1 + \omega'r_{T+1})$ in the absence of a risk-free asset and $W_{T+1} = W(1 + r_f + \omega'r_{T+1})$ when a risk-free asset with return $r_f$ is present. Let $\omega$ denote the vector of asset allocations (fractions of wealth allocated to the corresponding stocks). In a one-period setting, the optimal portfolio decision consists of choosing $\omega$ that maximizes the expected utility of next-period’s wealth,

$$\max_{\omega} E\left( U\left(W_{T+1}\right) \right) = \max_{\omega} \int U\left(W_{T+1}\right) p\left(r \mid \theta\right) dr,$$

subject to feasibility constraints, where $\theta$ is the parameter vector of the return distribution and $U$ is a utility function generally characterized by a quadratic or a negative exponential functional form. A key component of Eq. (24.1) is the distribution of returns $p\left(r \mid \theta\right)$, conditional on the unknown parameter vector $\theta$. The traditional implementation of the mean-variance framework proceeds with setting $\theta$ equal to its estimate $\hat{\theta}(r)$ based on some estimator of the data $r$ (often the maximum likelihood estimator). Then, the investor’s problem in Eq. (24.1) leads to the optimal allocation given by

$$\omega^* = \arg \max_{\omega} E\left[U\left(\omega'r\right) \mid \theta = \hat{\theta}(r)\right].$$

The solution in Eq. (24.2), known as the certainty equivalent solution, treats the estimated parameters as the true ones and completely ignores the effect of the estimation error on the optimal decision. The resulting portfolio displays high sensitivity to small changes in the estimated mean, variance, and covariance, and usually contains large long and short positions that are difficult to implement in practice.\(^2\)

Starting with the work of (Zellner and Chetty 1965), several early studies investigate the effect parameter uncertainty plays on optimal portfolio choice by re-expressing Eq. (24.1) in terms of the predictive density function.\(^3\) The predictive density function reflects estimation risk explicitly since it integrates over the posterior distribution, which summarizes the uncertainty about the model parameters.

---

1 The mean-variance selection rule of (Markowitz’s 1952), given by $\min \omega'\Sigma \omega$, s.t. $\omega'\mu \geq \mu^*, \omega'1 = 1$, where $\mu$ is the vector of expected returns, $\Sigma$ is the covariance matrix of returns, and $I$ is a compatible vector of ones, provides the same set of admissible portfolios as the quadratic-type expected-utility maximization in Eq. (23.1). (Markowitz and Usmen 1996) point out that the conventional wisdom that the necessary conditions for application of mean-variance analysis are normal probability distribution and/or quadratic utility is a “misimpression” (Markowitz and Usmen 1996, p. 217). Almost optimal solutions are obtained using a variety of utility functions and distributions. For example, it is possible to weaken the distribution condition to members of the location-scale family. See (Ortobelli, Rachev, and Schwartz 2004).

2 See, for example, (Best and Grauer 1991).

3 See, for example, (Barry 1974; Winkler and Barry 1975; Klein and Bawa 1976; Brown 1976; Jobson, Korkie and Ratti 1979; Jobson and Korkie 1980; Chen and Brown 1983).