Chapter 2
The Hungarian Method for the Assignment Problem

Harold W. Kuhn

Introduction by *Harold W. Kuhn*

This paper has always been one of my favorite "children," combining as it does elements of the duality of linear programming and combinatorial tools from graph theory. It may be of some interest to tell the story of its origin.

I spent the summer of 1953 at the Institute for Numerical Analysis which was housed on the U.C.L.A. campus. I was supported by the National Bureau of Standards and shared an office with Ted Motzkin, a pioneer in the theory of inequalities and one of the most scholarly mathematicians I have ever known. I had no fixed duties and spent the summer working on subjects that were of interest to me at the time, such as the traveling salesman problem and the assignment problem.

The Institute for Numerical Analysis was the home of the SWAC (Standards Western Automatic Computer), which had been designed by Harry Huskey and had a memory of 256 words of 40 bits each on 40 Williamson tubes. The formulation of the assignment problem as a linear program was well known, but a 10 by 10 assignment problem has 100 variables in its primal statement and 100 constraints in the dual and so was too large for the SWAC to solve as a linear program. The SEAC (Standard Eastern Automatic Computer), housed in the National Bureau of Standards in Washington, could solve linear programs with about 25 variables and 25 constraints. The SEAC had a liquid mercury memory system which was extremely limiting.

During that summer, I was reading König's book on graph theory. I recognized the following theorem of König to be a pre-linear programming example of duality:

If the numbers of a matrix are 0's and 1's, then the minimum number of rows and columns that will contain all of the 1's is equal to the maximum number of 1's that can be chosen, with no two in the same row or column.

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Indeed, the primal problem is the special case of an assignment problem in which the ratings of the individuals in the jobs are only 0's and 1's. In a footnote, König refers to a paper of E. Egerváry (in Hungarian), which seemed to contain the treatment of a more general case. When I returned to Bryn Mawr, where I was on the faculty in 1953, I took out a Hungarian grammar and a large Hungarian-English dictionary and taught myself enough Hungarian to translate Egerváry's paper. I then realized that Egerváry's paper gave a computationally trivial method for reducing the general assignment problem to a 0-1 problem. Thus, by putting the two ideas together, the Hungarian Method was born. I tested the algorithm by solving 12 by 12 problems with random 3-digit ratings by hand. I could do any such problem, with pencil and paper, in no more than 2 hours. This seemed to be much better than any other method known at the time.

The paper was published in Naval Research Logistics Quarterly. This was a natural choice since the project in Game Theory, Linear and Nonlinear Programming, and Combinatorics at Princeton, with which Al Tucker and I were associated from 1948 to 1972, was supported by the Office of Naval Research Logistics Branch. Many mathematicians were benificiaries of the wise stewardship of Mina Rees as head of the ONR and Fred Rigby as chief of the Logistics branch. We were also fortunate to have Jack Laderman, the first editor of the journal, as our project supervisor.

I have told much of the same story in my paper [1]. Large sections of this account are reproduced in the book by Alexander Schrijver [2]. Schrijver's account places the Hungarian Method in the mathematical context of combinatorial optimization and rephrases the concepts in graph-theoretical language.

References

1. H.W. Kuhn, *On the origin of the Hungarian Method*, History of mathematical programming; a collection of personal reminiscences (J.K. Lenstra, A.H.G. Rinnooy Kan, and A. Schrijver, eds.), North Holland, Amsterdam, 1991, pp. 77–81.
2. A. Schrijver, *Combinatorial optimization: polyhedra and efficiency*, Vol. A. Paths, Flows, Matchings, Springer, Berlin, 2003.