An Optimal Strategy for Online Non-uniform Length Order Scheduling*

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\textbf{Abstract.} This paper will study an online non-uniform length order scheduling problem. For the case where online strategies have the knowledge of $\Delta$ beforehand, which is the ratio between the longest and shortest length of order, Ting \cite{3} proved an upper bound of $(\frac{6\Delta}{\log \Delta} + O(\Delta^{5/6}))$ and Zheng et al. \cite{2} proved a matching lower bound. This work will consider the scenario where online strategies do not have the knowledge of $\Delta$ at the beginning. Our main work is a $(\frac{6\Delta}{\log \Delta} + O(\Delta^{5/6}))$-competitive optimal strategy, extending the result of Ting \cite{3} to a more general scenery.

\textbf{Keywords:} Scheduling, Online Strategy, Competitive Ratio.

1 Introduction

Since there are many dynamic and unpredictable factors in modern manufacturing business, including production order scheduling and processing operations, many authors adopt online theory to describe and investigate the scenario, and online scheduling has caught much interest among extensive scheduling literature in recent decades. In online scheduling, there is a manufacturer who may accept or decline orders that arrive one by one over time. Each order will stay in the system to be satisfied after arrival until it expires, i.e., its deadline cannot be met at the time even started at once. The manufacturer will gain a profit from each completed order. He may also abort an order on running to start a new one in favor of larger profit, and the aborted order has to be started again from the beginning to be satisfied. That is, we consider the preemption-restart online model to maximize the total profit of completed orders.

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In most literature, it is usually assumed that the shortest and longest order is of one unit and $\Delta$ units of length respectively. Most literature studied the case where the knowledge of $\Delta$ is known to online strategies at the first beginning. Fung et al. [1] studied an online broadcast problem. Translated into our terminology, they proved a $\Delta + 2\sqrt{\Delta} + 2$-competitive strategy ACE (Another Completes Earlier). Zheng et al. [2] present a lower bound of $\Omega(\Delta/\ln \Delta)$. Ting [3] proved a matching upper bound of $O(\Delta/\log \Delta)$, and left it open whether there exist $O(\Delta/\log \Delta)$-competitive online strategies without the knowledge of $\Delta$. In the above studies, online strategies make use of $\Delta$. Kim et al. [4] presented a 5-competitive greedy strategy GD for the case of unit length of order, i.e., $\Delta = 1$. GD makes an abortion if a newly arrival order has weight $\alpha$ times that of the order being processed, otherwise GD continues the current service. Zheng et al. [5] proved that GD is $4\Delta + 1$-competitive with $\alpha = 2$ for the case where $\Delta \geq 1$. GD is treated as an online strategy that acts without the knowledge of $\Delta$ and it performs poorly for the case where $\Delta > 1$. For the other case where $\Delta$ is unknown beforehand, it is harder for online strategies to act efficiently, and to the best of our knowledge there is little result considering to maximize profits. We believe that it is a reasonable case in real business since the manufacturer usually can not foresee the exact information of future orders, including order length. So, the work will focus on the performance of online strategies in the latter case and give a positive answer to the problem proposed in Ting [3].

We describe our problem formally as follows. There is one manufacturer who processes orders arriving over time. Each order $J$ has four attributes, namely, $a(J)$: the arrival time, $p(J)$: the processing time or the length, $w(J)$: the profit or weight to be obtained only if $J$ is finished and $d(J)$: the deadline by which the order has to be completed to be satisfied. $p(J)$, $w(J)$ and $d(J)$ become known on the arrival of $J$, i.e., at time $a(J)$. $1 \leq p(J) \leq \Delta$ where $\Delta$ is assumed w.l.o.g. to be a natural number. The goal is to maximize the total profit of completed orders within a time period.

### 1.1 Related Work

One related direction is the scenario where $p(J)$ is unknown to online strategies until order $J$ has been completed. The scenery is called *non-clairvoyant scheduling* and was first investigated by Motwani et al [7]. They aimed to minimize the total flow time, presenting a lower bound of $\Omega(n^{1/3})$ for deterministic non-clairvoyant strategies and of $\Omega(\log n)$ for randomized ones where $n$ is the number of orders, respectively. Kalyanasundaram and Pruhs [8] proposed a $O(\log n \log \log n)$-competitive randomized strategy RMLF against a adaptive adversary. Becchetti and Leonardi [9] further showed that the RMLF strategy is in fact $O(\log n)$-competitive, matching the lower bound.

Another quite related line is the scenario where the length of order is selected from a finite set of real numbers but not an arbitrary number within $[1, \Delta]$. Lipton and Tomkis [10] studied the scenario to maximize resource utilization. One of their results is an upper bound of 2 for non-preemption strategies in the case where $p(J)$ is either 1 or $\Delta$ and $d(J) = a(J) + p(J)$. Goldwasser [11] extended