An Overview on the Split Delivery Vehicle Routing Problem

Claudia Archetti and Maria Grazia Speranza

Department of Quantitative Methods, University of Brescia, Italy
archetti@eco.unibs.it
speranza@eco.unibs.it

Summary. In the classical Vehicle Routing Problem (VRP) a fleet of capacitated vehicles is available to serve a set of customers with known demand. Each customer is required to be visited by exactly one vehicle and the objective is to minimize the total distance traveled. In the Split Delivery Vehicle Routing Problem (SDVRP) the restriction that each customer has to be visited exactly once is removed, i.e., split deliveries are allowed. In this paper we present a survey of the state-of-the-art on this important problem.

1 Introduction

We consider the *Split Delivery Vehicle Routing Problem* (SDVRP) where a fleet of capacitated homogeneous vehicles has to serve a set of customers. Each customer can be visited more than once, contrary to what is usually assumed in the classical *Vehicle Routing Problem* (VRP), and the demand of each customer may be greater than the capacity of the vehicles. There is a single depot for the vehicles and each vehicle has to start and end its tour at the depot. The problem consists in finding a set of vehicle routes that serve all the customers such that the sum of the quantities delivered in each tour does not exceed the capacity of a vehicle and the total distance traveled is minimized.

The SDVRP was introduced in the literature only a few years ago by Dror and Trudeau ([13] and [14]) who motivate the study of the SDVRP by showing that there can be savings generated by allowing split deliveries. Archetti, Savelsbergh and Speranza [3] study the maximum possible savings obtained by allowing split deliveries, while in [4] the same authors present a computational study to show how the savings depend on the characteristics of the instance. Valid inequalities for the SDVRP are described in [12]. In [9] a lower bound is proposed for the SDVRP where the demand of each customer is lower than the capacity of the vehicles and the quantity delivered by a vehicle when visiting a customer is an integer number. In [2] the authors analyze the computational complexity of the SDVRP and the case of small capacity of the vehicles.

Heuristic algorithms for the SDVRP can be found in [13] and [14], where a local search algorithm is proposed, in [1] for a tabu search and in [5] for an optimization-

based heuristic. In [15] the authors present a mathematical formulation and a heuristic algorithm for the SDVRP with grid network distances and time windows constraints.

Real applications of the problem can be found in [18] where the authors consider the problem of managing a fleet of trucks for distributing feed in a large livestock ranch which is formulated as a split delivery capacitated rural postman problem with time windows. Several heuristics are proposed to solve the problem which compare favorably with the working practices on the ranch. Sierksma and Tijssen [19] consider the problem of determining the flight schedule for helicopters to off-shore platforms for exchanging crew people employed on these platforms. The problem is formulated as an SDVRP and several heuristics are proposed. In [6] Archetti and Speranza consider a waste collection problem where vehicles have a small capacity and customers can have demands larger than the capacity. A number of constraints are considered like time windows, different types of wastes, priorities among customers and different types of vehicles. They propose a heuristic algorithm that beats the solution implemented by the company which carries out the service. A similar problem is analyzed in [8], where it is called the Rollon-Rolloff Vehicle Routing Problem (RRVRP), and in [11].

A detailed survey of the state-of-the-art on the split delivery vehicle routing problem can be found in [7].

The paper is organized as follows. In Section 2 we provide the problem description and we present computational complexity results and some properties of the problem. In Section 3 we analyze the savings with respect to the VRP and a simple heuristic for the VRP and the SDVRP. In Section 4 we present the heuristic algorithms proposed for the SDVRP and compare them.

2 The Split Delivery Vehicle Routing Problem

The SDVRP can be defined over a graph $G = (V,E)$ with vertex set $V = \{0,1,...,n\}$ where 0 denotes the depot while the other vertices are the customers, and E is the edge set. The traversal cost (also called length) c_{ij} of an edge $(i,j) \in E$ is supposed to be non-negative and to satisfy the triangle inequality. An integer demand d_i is associated with each customer $i \in V - \{0\}$. An unlimited number of vehicles is available, each with a capacity $Q \in Z^+$. Each vehicle must start and end its route at the depot. The demands of the customers must be satisfied, and the quantity delivered in each tour cannot exceed Q. The objective is to minimize the total distance traveled by the vehicles. A mixed integer programming formulation for the SDVRP is provided in [1].

In [2] it is shown that the SDVRP with $Q = 2$ can be solved in polynomial time, while it is NP-hard for $Q \geq 3$.

Dror and Trudeau [13] have shown an interesting property of optimal solutions to the SDVRP. To understand their result we first need the following definition.

Definition 1. *Consider a set $C = \{i_1,i_2,...,i_k\}$ of customers and suppose that there exist k routes $r_1,...,r_k$, $k \geq 2$, such that r_w contains customers i_w and i_{w+1}, $w = 1,...,k-1$, and r_k contains customers i_1 and i_k. Such a configuration is called a k-split cycle.*