Scheduling Buses and School Starting Times

Armin Fügenschuh

Fachgebiet Optimierung, Technische Universität Darmstadt, Germany
fuegenschuh@mathematik.tu-darmstadt.de

1 Introduction

Traffic peaks are peaks in cost. This in particular holds for rural counties, where the organization of public mass transportation is focused on the demand of pupils. About half to two third of pupils in rural areas take a bus to get to school. Most of them are integrated in the public bus system, a minority is transfered by special purpose school buses. In all cases the respective county in which the pupils live is responsible for the transfer, meaning that the county administration pays the fees. Since tax money is a scarce resource, the administration has great interest in reducing these payments.

A significant number of buses could be saved, if the bus scheduling problem is solved together with the starting time problem, i.e., the simultaneous settlement of school and trip starting times [6, 7]. A small intuitive example is shown in Figure 1. If two schools start at the same time then two different buses are necessary to bring the pupils to their respective schools. If they start at different times then one and the same bus can first bring pupils to one school and then pupils to the other. In this article we describe how to roll out this intuitive idea to a whole county. Besides presenting a mathematical formulation in Section 2 and computational results in Section 5, we want to

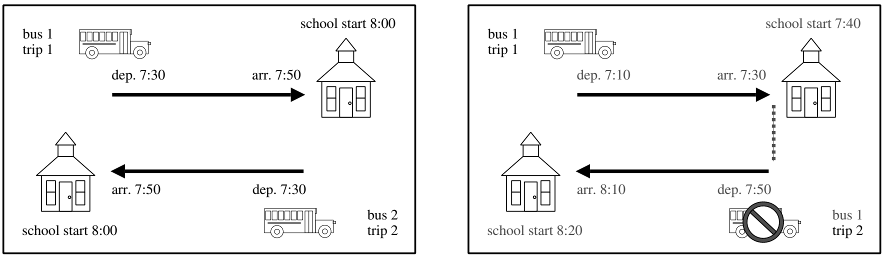

Fig. 1. The central idea (before – after)

put a particularly emphasis on the "legacy" of the PhD thesis [2]. That is, we want to point out those parts that are suitable for solving other combinatorial optimization problems: A new metaheuristic in Section 3 and a new preprocessing scheme in Section 4.

2 A Mathematical Model

Let $\mathcal{V}$ be the set of all passenger trips in the county under consideration. A *passenger trip* (or *trip* for short) $t \in \mathcal{V}$ is a sequence of bus stops, each having an arrival and a departure time assigned to. The time difference between the departure at the first and the arrival at the last bus stop is called the *trip duration*, and is denoted by δ_t^{trip}. (All time-related parameters and variables in this model are integral with the unit "minute".) For every trip $t \in \mathcal{V}$ we introduce an integer variable $\alpha_t \in \mathbb{Z}_+$ representing its planned starting time, i.e., the departure of a bus at the first bus stop. A time window $\underline{\alpha}_t, \overline{\alpha}_t$ is given, in which the planned trip starting time must be, see inequalities (1f).

All buses start and end their tours at a *depot*. The trip to the first bus stop of trip t is called *pull-out trip*. When the bus arrives at the last bus stop of t, it is either sent on the *pull-in trip* back to the depot, or it serves another passenger trip. The duration of the pull-out and pull-in trip is denoted by $\delta_t^{\text{out}}, \delta_t^{\text{in}}$, respectively. The intermediate trip from the last bus stop of trip t_1 to the first bus stop of trip t_2 is called *deadhead trip*. The connection of a pull-out trip, several passenger and deadhead trips and a final pull-in trip which are then served by one and the same bus is called a *schedule*. For every trip $t \in \mathcal{V}$ the decision variables $v_t, w_t \in \{0,1\}$ indicate if trip t is the first or the last trip in some schedule, respectively. Let the set $\mathcal{A} \subset \mathcal{V} \times \mathcal{V}$ contain all pairs of trips (t_1, t_2) that can in principle be connected by a deadhead trip. The duration of the deadhead trip is given by $\delta_{t_1 t_2}^{\text{shift}}$. For every pair of trips $(t_1,t_2) \in \mathcal{A}$ the variable $x_{t_1 t_2} \in \{0,1\}$ indicates if t_1 and t_2 are in sequence in some schedule, that is, the same bus serves trip t_2 directly after finishing trip t_1 (apart from the deadhead trip and the idle time). Each trip is served by exactly one bus which is ensured by assignment constraints (1b). If trips $(t_1,t_2) \in \mathcal{A}$ are connected, then trip t_2 can only start after the bus has finished trip t_1, shifted from the end of t_1 to the start of t_2. Additional waiting is permitted if the bus arrives before the start of t_2. Using a sufficiently big value for M, these constraints can be formulated as linear inequalities (1c).

Let $\mathcal{S}$ be the set of all schools in the county. For every school $s \in \mathcal{S}$ we introduce an integer variable $\tau_s \in \mathbb{Z}_+$. The school starting time is required to be in discrete time slots of 5 minutes (7:30, 7:35, 7:40, etc.). Thus the planned starting time of s is $5 \cdot \tau_s$. It is allowed to change this starting time within some time window $\underline{\tau}_s, \overline{\tau}_s$. Usually, the time window constraints (1f) reflect the legal bounds on the school starting time (7:30 to 8:30 a.m.).

The set $\mathcal{P} \subset \mathcal{S} \times \mathcal{V}$ consists of pairs (s,t), where trip t transports pupils to a bus stop of school s. In this case we say t is a *school trip* for s. The time