A Multi-Commodity Flow Approach for the Design of the Last Mile in Real-World Fiber Optic Networks*

Daniel Wagner[1], Günther R. Raidl[2], Ulrich Pferschy[3], Petra Mutzel[4], and Peter Bachhiesl[1]

[1] Department of Telematics and Network Engineering, Carinthia University of Applied Sciences, Austria
d.wagner@fh-kaernten.at
p.bachhiesl@fh-kaernten.at

[2] Institute of Computer Graphics and Algorithms, Vienna University of Technology, Austria
raidl@ads.tuwien.ac.at

[3] Institute of Statistics and Operations Research, University of Graz, Austria
pferschy@uni-graz.at

[4] Chair for Algorithm Engineering, University of Dortmund, Germany
petra.mutzel@cs.uni-dortmund.de

Summary. We consider a generalization of the Steiner tree problem on graphs suitable for the design of the last mile in fiber optic networks and propose a multi commodity flow formulation for the exact solution of this problem. Some experimental results are discussed.

1 Introduction

We consider the problem of finding a most cost-efficient fiber optic network to connect given customer nodes to an existing network infrastructure.

Given is a connected undirected graph $G = (V,E,c,l,p,k_{\max})$ describing the topology of the surrounding area of given customer nodes. Each edge $e = (i,j) \in E$ represents a straight segment of Euclidean length $l_e \geq 0$ where a fiber optics cable might be installed with construction costs $c_e \geq 0$. The existing infrastructure is given as a subgraph $I = (V_I, E_I)$ of G. Furthermore the set of nodes V consists of the customer nodes C that shall be connected and spatial nodes S (possible Steiner nodes) resulting from the underlying spatial topology. The customer set C is the disjoint union of C_1 and C_2, whereby customers C_1 require a single connection and customers C_2 need to be redundantly connected.

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In real-world network design we have to consider several technical and practical requirements which make the problem more difficult. We model them by the following constraints.

- *Junction constraint:* Customer nodes of set C have to be connected to the existing infrastructure. Attaching new connections to the infrastructure is only allowed at predefined junction nodes $J \subseteq V_I$.
- *Biconnectivity constraint:* Customer nodes of set C_2 need to be redundantly connected to the existing infrastructure by two node-disjoint paths.
- $k_{\max}$*-redundancy constraint:* Occasionally, the biconnectivity constraint for the nodes in the set C_2 may be relaxed in the sense that such a node may be connected to any biconnected (Steiner or customer) node v via a single path of maximum Euclidean length $k_{max}(k)$.
- *Non-crossing constraint:* Cable routes are not allowed to cross each other in a geometric sense.

The *Operative Planning Task* (OPT) consists of finding a minimum-cost connected subgraph $G' = (V_{G'}, E_{G'})$ of G, $V_{G'} \subseteq V$, $E_{G'} \subseteq E$ that connects all customers so that the technical constraints are satisfied.

An extended variant is the *Strategic Simulation Task* (SST) that represents a generalization of the prize-collecting Steiner tree problem [5]. For each customer $c \in C$ a prize $p_c \geq 0$, i.e. an intended return on investment, is introduced. The problem is to find a subgraph G' of G that connects a subset of customers, minimizing

$$\sum_{e \in E_{G'}} c_e + \sum_{i \in C \setminus V_{G'}} p_i$$

so that the technical constraints are satisfied as in the OPT variant.

2 An Approach Based on Multi Commodity Flows

Multi commodity flow formulations are known to yield strong descriptions for many network design tasks including various constrained spanning and Steiner tree problems [1, 3, 6]. The basic idea is to send in G one commodity from the root node 0, which results from shrinking the infrastructure I into a single node, to each customer node $k \in C$ to be connected. This already satisfies the junction constraint.

To realize the biconnectivity constraint for each customer node $k \in C_2$, two different commodities f^k and g^k are sent in G from the root node 0 to the customer node k, which may not share any edges or have nodes other than the root 0 and k in common. With respect to the $k_{\max}$-redundancy constraint we introduce an auxiliary commodity h^k in the $k_{\max}$-neighborhood of customer k that indicates the segment where commodities f^k and g^k flow along the same path according to the definition of $k_{\max}$-redundancy.

We define the following node, edge and arc sets.

- Arcs connecting the root with spatial nodes: $A_0 = \{(0,j) \in E \mid j \in S\}$
- Edges connecting two spatial nodes with respect to customer $k \in C$: $E_S(k) = \{(i,j) \mid i,j \in V \setminus \{0,k\}\}$
- Pairs of reversely directed arcs corresponding to the edges in $E_S(k)$: $A_S(k) = \{(i,j), (j,i) \mid (i,j) \in V \setminus \{0,k\}\}$.