Polynomial Algorithms for Some Hard Problems of Finding Connected Spanning Subgraphs of Extreme Total Edge Weight

Alexey Baburin and Edward Gimadi

Sobolev Institute of Mathematics SB RAS, pr. Koptyga 4, Novosibirsk, Russia
ababur@sicex.ru
gimadi@math.nsc.ru

Summary. Several hard optimization problems of finding spanning connected subgraphs with extreme total edge weight are considered. A number of results on constructing polynomial algorithms with performance guarantees for these problems is presented. [1]

1 Introduction

This paper is devoted to address a subject of several previous works by the authors on problems of finding a restricted spanning subgraph of extreme total edge weight in a given graph. Restrictions imply connectivity of the resulting subgraph and predefined values of degrees of its vertices.

One of the most popular problems of the kind (when the degrees of all vertices equal two) is TSP [16]. The problem is MAX SNP-hard: existence of a polynomial approximation scheme for it yields $P = NP$. The Euclidean TSP in k-dimensional Euclidean space is also NP-hard. Although the status of the problem for $k = 2$ remains unclear. However there exist polynomial asymptotically optimal algorithms for solving the deterministic Maximum Euclidean TSP [2, 15]. We imply different restrictions on the resulting subgraph thus considering several generalizations of TSP.

We present polynomial approximation algorithms with performance guarantees for the problem of finding a connected spanning subgraph of maximum total edge weight with given (arbitrary) vertex degrees. Performance guarantees are proved for three cases of edge weights of the input graph: arbitrary non-negative weights, weights satisfying triangular inequality, Euclidean distances [10], [4]. The problem of finding a regular connected subgraph of extreme total edge weight in a complete graph when the weights of the edges are random variables is also considered. Polynomial asymptotically optimal algorithm is presented for maximization and minimizations variants of the problem [4].

[1] Research was supported by Russian Foundation for Basic Research (project 05-01-00395) and INTAS (grant 04-77-7173).

Another problem considered is a problem of finding several edge-disjoint Hamiltonian circuits of extreme total edge weight. The first mention of the problem called the m-Peripatetic Salesman Problem (m-PSP) in the literature came in [13]. Recently we developed several polynomial algorithms with performance guarantees for solving the maximum 2-PSP, and the metric minimum 2-PSP (cases of equal and independent weight functions of edges for measuring the weights of two Hamiltonian circuits constructed) [1], [3].

We also consider a maximization variant of Euclidean m-PSP and outline an approximation algorithm for solving it.

2 Finding a Spanning Connected Subgraph of Extreme Total Edge Weight and Given Vertex Degrees

Let $G(V, E)$ be a complete n-vertex undirected graph without loops with a non-negative weight function w of edges. There are known integers d_i $(i = 1, \ldots, n)$, $1 \leq d_i < n$.

In [12] the problem of finding a realization of a set of integers as degrees of the vertices in a subgraph G' of G is formulated. (This set is called a graphical partition of a number p, where $p = \sum_{i=1}^{n} d_i$). It is clear that for every such realization the number p is even and $d_i < n$ for each $i = 1, \ldots, n$. These conditions are not sufficient. A constructive criterion of realizability of a set of integers was developed in [11].

An optimization appearance of the problem was described in [8] by means of reduction to the problem of finding a maximum-weight matching.

The problem of finding a maximum-weight spanning connected subgraph with given vertex degrees appeared in [10]. An approximation algorithm was presented for the metric case of the problem. The algorithm only worked for the case when all integers d_i were even. In [4] a new approximation algorithm was developed. It did not require d being even and worked for general case of the problem.

Theorem 1. *[4] An approximate solution for the problem can be found with running-time $O(Mn^2)$, where M is the number of edges in the subgraph being built. The performance ratio of the solution does not exceed $2/(d^2 + d)$, where $d = min\{d_i | i = 1, \ldots, n\}$. In the metric case of the problem the performance ratio does not exceed $1/(d^2 + d)$.*

3 Finding d-regular Connected Subgraph in a Complete Graph with Random Edge Weights

Let $G(V, E)$ be a complete n-vertex undirected graph without loops with a weight function $w : E \to R^+$ of edges and d, $1 < d < n$, be a given number. The problem is to find a d-regular connected subgraph $\widetilde{G}$ of G of maximal (or minimal) total edge weight. A solution always exists for the case of even nd and $d < n$. The case of $d = 2$ is the Maximum Traveling Salesman Problem. A survey of results on the later problem can be found in [16].