Using Shadow Prices to Reveal Personal Preferences in a Two-Stage Assignment Problem

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1 Introduction

Many events such as conferences or informational events at institutions consist of a number of single workshops among which participants can choose a limited number to visit. The workshop assignment problem is a linear optimization problem that finds a feasible assignment of participants to workshops maximizing the social welfare function expressed by preferences of participants for workshops. Several problem constraints reflect the structure and limitations of the event and define possible workshop combinations.

The optimization problem is formulated based on a priori preferences of participants over the workshops. Preferences constitute private information. For several reasons, preferences may be interrelated and can thus be influenced by the preferences of other participants and the optimal assignment resulting from them. The publication of the optimal assignment reveals additional information to the participants that may influence their initial preferences. As a result, the assignment is no longer optimal with respect to the modified preferences which has a negative effect on the satisfaction of the participants with the assigned workshops.

To overcome this problem, we introduce a second optimization stage. During the second stage, participants can use the additional information revealed by the publication of the first assignment to modify their preferences. Shadow prices are used to communicate the necessary information in an efficient way. Based on the new preference values and a slightly modified problem definition, possible changes in the current assignment are calculated. These changes are restricted in such a way that no participant may be assigned a combination that is worse compared to his currently assigned workshops. Simulation results show that the so-defined second stage can effectively be used to improve the assignment and that shadow prices are a good means for providing the necessary information to the participants.

2 The Workshop Assignment Problem

The workshop assignment problem is formulated as a binary, linear maximization problem. It allows for workshop capacity restrictions, time restrictions on workshops as well as participants and specification of workshop equivalence. Let $\mathbb{B} = \{0,1\}$. Let $T = \{1, \ldots, \overline{t}\}$ denote the timeslots or indivisible time units of the event. Let $W = \{1, \ldots, \overline{w}\}$ denote the set of workshops where each workshop takes place exactly once. Workshops taking place multiple times are modelled as several equivalent workshops by the equivalence relation $\sim$. Participants should attend at most one workshop of each equivalence class where $[w]_\sim$ is the equivalence class of workshop $w \in W$, and $W/\sim$ is the set of all such classes. The positive entries in the matrix $\mathbf{Q} \in \mathbb{B}^{\overline{w}\times\overline{w}}$ specify equivalent workshops. $P = \{1, \ldots, \overline{p}\}$ is the set of the participants. The capacity limitations of the workshops are given by the vector $\mathbf{c} \in \mathbb{N}^{\overline{w}}$. Each participants visits a minimum number of workshops, given by the vector $\mathbf{m} \in \mathbb{N}^{\overline{p}}$. The matrix $\mathbf{F} \in \mathbb{B}^{\overline{w}\times\overline{t}}$ is an interval matrix where the 1-entries specify the timeslots occupied by each workshop. The presence of the participants in the timeslots is given by the matrix $\mathbf{B} \in \mathbb{B}^{\overline{p}\times\overline{t}}$. The participants' preferences for the workshops are given by the matrix $\mathbf{S} \in \mathbb{R}^{\overline{p}\times\overline{w}}$. Preferences are specified using a partial preference function. A default preference value is assigned to all workshops that are not explicitly given a preference value by a participant. Preference values smaller than the default preference mark workshops that the participant does not wish to attend whereas higher values stand for a positive preference. Preferences are chosen by the participants from a pool of available preference values. This pool may contain the same preference value more than once, workshops with the same preference assigned are considered to be equally desirable for the participant.

With $x_{pw} \in \mathbb{B}$ denoting the assignment of participant p to workshop w, the workshop assignment problem is formulated as follows.

$$\max \textstyle\sum_{p\in P, w\in W} s_{pw} \cdot x_{pw} \tag{1}$$

subject to

$$\textstyle\sum_{p\in P} x_{pw} \leq c_w \qquad \forall w \in W \tag{2}$$

$$x_{pw} = 0 \qquad \forall p \forall w (p \in P \wedge w \in W \wedge \exists t (t \in T \wedge f_{wt} = 1 \wedge b_{pt} = 0)) \tag{3}$$

$$\textstyle\sum_{v\in[w]_\sim} x_{pv} \leq 1 \qquad \forall p \forall w (p \in P \wedge w \in W \wedge |[w]_\sim| > 1) \tag{4}$$

$$\textstyle\sum_{w\in W} x_{pw} \geq m_p \qquad \forall p \in P \tag{5}$$

$$\textstyle\sum_{w\in W: f_{wt}=1} x_{pw} \leq 1 \qquad \forall p \in P, \forall t \in T \tag{6}$$

$$x_{pw} \in \{0,1\} \qquad \forall p \in P, \forall w \in W \tag{7}$$

Condition (2) ensures that the capacity limitations are met, (3) prohibits to assign participants to workshops that take place while they are not present, (4) prevents the assignment of one participant to more than one workshop from an equivalence class, (5) makes sure that each participant visits his required minimum number of workshops and finally (6) ensures that no two workshops are assigned that overlap in time.

3 Preference Dependencies

Why and how do preferences of participants depend on each other? There are several possible causes for such dependencies.