A Coherent Spot/Forward Price Model with Regime-Switching

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1 Introduction

The challenge in modelling electricity prices is mainly caused by it's non-storability. Spot prices are thus determined by the current demand/supply interaction, but hardly by expectations about the future. They show characteristics as mean-reversion, seasonal patterns, an immense volatility and spikes, which cannot be captured with standard stock market models. On contrary, there exists growing markets, where financial futures contracts are traded. These contracts are storable and show similar characteristics to other financial assets. In particular they feature a significant lower volatility then spot prices. Moreover, the volatility is decreasing in the time to maturity.

Hence, when modelling electricity prices one is actually confronted with two classes of prices, which show quite different characteristics. In the field of electricity price modelling there exist then also a number of models, which either focus on spot or on futures prices. However there are only few approaches, which try to capture the features of both prices in one model.

In this work we formulate and estimate a joint model, which describes spot and futures prices for electricity, taking the special characteristics of both prices into account.

2 Formulation of the Model

We consider an economy in which uncertainty is modelled by the filtered probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\mathcal{P}$ the real world probability measure. Events are revealed over time according to the filtration $\mathcal{F} = \{\mathcal{F}_t\}_{t\in[0,T^*]}$, where T^* is a fixed time horizon, which limits the considered trading interval $[0, T^*]$. The market model contains electricity futures prices $F(t,T)$ of different maturities T as the prices of primary traded assets, where $0 < T < T^*$ and $t \leq T$. In addition it contains a riskless instrument. For simplicity of exposition, we assume that the risk-free interest rate is zero. The model also specifies the spot price process S_t, which is considered as a non-traded asset. We further assume that the market is free of arbitrage and that

there exists an equivalent probability measure $\mathcal{Q}$, under which futures prices are martingales.

The market security prices are assumed to be functions of k state variables $X_t = (X_t^1,...,X_t^k)'$. We consider two model specifications. The first model, called the basis model, is a two factor diffusion model, where X has the following dynamics under $\mathcal{P}$:

$$\begin{aligned} dX_t^1 &= (\kappa_1(X_t^2 - X_t^1))dt + \sigma_1 dW_t^1 \\ dX_t^2 &= \mu_2 dt + \sigma_2 dW_t^2 \ . \end{aligned} \tag{1}$$

X^1 captures the short term movements of electricity prices. It is a mean-reverting process. The mean reversion level can be interpreted as the equilibrium price level, which is modeled as an arithmetic Brownian motion. The two Wiener processes W^1 and W^2 are independent. We assume further constant market prices of risk ϕ_1, ϕ_2, such that the SDE's of the X^i under $\mathcal{Q}$ differ just by a constant reduction of the drift. The spot price S_t is composed of a seasonality factor $N(t) \equiv e^{n(t)}$ and the exponential of a weighted sum of the state variables:

$$S_t = N(t)\mathrm{e}^{\gamma' X_t} \ . \tag{2}$$

In the basis model we choose $\gamma = (1,0)'$. Assuming an arbitrage-free futures market, the futures price $F(t,T)$ must be a $\mathcal{Q}$-martingale. Furthermore, the absence of arbitrage opportunity implies that it must converge towards the spot price at maturity. Thus the futures price is given as:

$$\begin{aligned} F(t,T) &= E_t^Q[S_T] = N(T)E_t^Q[e^{\gamma' X_T}] \\ &= N(T)\exp\Big((-\tfrac{\mu_2-\phi_2}{\kappa_1} - \tfrac{\phi_1}{\kappa_1})(1-\mathrm{e}^{-\kappa_1(T-t)}) + (\mu_2-\phi_2)(T-t) \\ &\quad + \tfrac{\sigma_1^2+\sigma_2^2}{4\kappa_1}(1-\mathrm{e}^{-2\kappa_1(T-t)}) + \tfrac{\sigma_2^2}{2}(T-t) - \tfrac{\sigma_2^2}{\kappa_1}(1-\mathrm{e}^{-\kappa_1(T-t)}) \\ &\quad + \mathrm{e}^{-\kappa_1(T-t)}X_t^1 + (1-\mathrm{e}^{-\kappa_1(T-t)})X_t^2\Big) \\ &= N(T)\exp(\alpha(t) + \beta(t)X_t) \ . \end{aligned} \tag{3}$$

Applying Ito's-Lemma yields:

$$\frac{dF_t}{F} = \mathrm{e}^{-\kappa_1(T-t)}\sigma_1 d\tilde{W}_t^1 + (1-\mathrm{e}^{-\kappa_1(T-t)})\sigma_2 d\tilde{W}_t^2 \ , \tag{4}$$

where $\tilde{W}^1$ and $\tilde{W}^2$ are Wiener processes under $\mathcal{Q}$. $\tilde{W}^1$ accounts for the fluctuations of futures prices with short maturities, whereas the second determines the long-term fluctuations. Thus there is no perfect correlation between futures prices of different maturities.

The second model we consider, the regime-switching model, is an extension of the first one. A third risk-factor X^3 is introduced. It is a pure jump process, which is modeled as a two state process with states J and K. The state variables X are then described by the following dynamics under $\mathcal{P}$:

$$\begin{aligned} dX_t^1 &= (\kappa_1(X_t^2 - X_t^1))dt + \sigma_1 dW_t^1 \\ dX_t^2 &= \mu_2 dt + \sigma_2 dW_t^2 \\ dX_t^3 &= 1_{X_t^3=0}d\nu_t^J - X_t^3 d\nu_t^K \ . \end{aligned} \tag{5}$$

$1_{X_t^3=0}$ denotes the indicator function. Thus, the process X^3 can jump into the spike state J only if it is in the normal state K, and it can jump back only, when it is in