Heuristic Optimization of Reinsurance Programs and Implications for Reinsurance Buyers

Andreas Mitschele[1,2], Ingo Oesterreicher[1], Frank Schlottmann[2], and Detlef Seese[1]

[1] Institute AIFB, University of Karlsruhe, Germany
mitschele@aifb.uni-karlsruhe
deseese@aifb.uni-karlsruhe.de
IngoOest@gmx.de
[2] GILLARDON AG financial software, Bretten, Germany
frank.schlottmann@gillardon.de

1 Introduction

Reinsurance contracts represent a very important tool for insurance companies to manage their risk portfolio. In general, they are used if an insurer is not willing or not able to hold certain risk exposures or parts thereof on its own. There exist two main contract types to cede claims to a reinsurer, namely proportional and non-proportional ones. With the *quota share reinsurance*, a well-known variant of the former ones, a fixed percentage of the claim sizes is ceded to the reinsurance company. *Excess of loss* and *stop loss* are non-proportional types and the reinsurer is only liable to pay if certain losses are exceeded. In practice insurance companies usually place a number of different reinsurance contracts, a so-called *reinsurance program*.

While a substantial amount of research has been performed to optimize the terms and conditions of individual reinsurance contracts separately, the simultaneous optimization within a reinsurance program has hardly been analyzed to date and only few results exist so far[3]. Even though an integrated view of the reinsurance program is highly desirable, mathematical intractability, computational complexity and non-convexity of the objective functions have previously impeded the realization of this goal. In this contribution we therefore introduce a novel multi-objective approach to fulfil the task of optimizing reinsurance contracts simultaneously. Thus we minimize the expenses that come with contracting a number of different reinsurance protections and at the same time we minimize the retained risk after reinsurance.

[3] Centeno [4] for instance focuses on combinations of quota share and excess of loss reinsurance, considering different cost calculation principles and risk measures. For a recent overview of current research see Verlaak/Beirlant [7] who also restrict their analysis to combinations of two reinsurance contracts.

2 Reinsurance Business and Optimization

In this section we briefly introduce some basic concepts from insurance mathematics[4] and give an introduction to reinsurance contract optimization.

We consider three important types of reinsurance in this contribution. The simplest form is the *quota share* (QS), where a fixed relative amount $a \in [0;1]$ of the claims and the premium income is ceded. In the other two types claims are cut at a positive line called priority. The *excess of loss* (XL) reinsurance cuts each individual claim at the priority R. The *stop loss* (SL) reinsurance cuts the sum of all claims in a year of business at the priority L. All contract types protect insurers from deviations in claim frequency and severity, however in complementary ways. Thus we contemplate combinations of two and three of these in our empirical investigation.

Reinsurance pricing is done via the expected value of the amount of claims ceded to the reinsurance company $\overline{S}$, i.e. $(1+\lambda)E(\overline{S})$[5]. The risk of the retained net sum of claims $\underline{S}$ is assessed using three different measures, namely the variance, the common risk measure *Value at Risk* (VaR_α) and the more advanced *Conditional Value at Risk* (CVaR_α)[6]. VaR_α is denoted by the α-quantile of the distribution of $\underline{S}$ while CVaR_α represents the expectation of the conditional distribution below the α-quantile.

For reinsurance optimization we use a modified *Mean-Variance-Criterion* similar to the one used in [7]. As objective functions we choose the minimization of the cost of $\overline{S}$ and the minimization of the risk of $\underline{S}$[7]. Employing a heuristic approach has several advantages compared to the standard approach using the Lagrange method of multipliers. We do not need to transform to a single objective function. Moreover, our model incorporates discrete step sizes which is more realistic than a continuous framework. Centeno [4] proves that even in simple problem instances the feasible search space is not convex in general. Verlaak/Beirlant [7] state that the computation of optimal reinsurance contracts for a combination of XL and SL is an open problem[8].

3 MOEA Approach

For the first time we treat the problem like a true multi-objective optimization problem and apply two state-of-the-art *multi-objective evolutionary algorithms* (MOEAs)[9] to obtain risk/return efficient reinsurance contract allocations.

Hereby we pursue two main goals: Firstly, we intend to solve reinsurance optimization problems for cases which still pose open problems to researchers. Secondly, our aim is to find a converged and diverse but dense front of Pareto optimal solutions in a single run to enable the decision maker to choose the desired relationship

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[4] For a comprehensive introduction we refer to [1].

[5] The factor λ describes the risk loading and depends on the reinsurance type.

[6] Also known as *Expected Shortfall* (ES).

[7] The distribution of the claims is based on real world data.

[8] This problem of computational complexity is caused by the need to compute the distribution of the sums $\underline{S}$ and $\overline{S}$ numerically for each combination of R and L.

[9] We compare NSGA-II [2] and ε-MOEA [3]. Details of chosen parameters cf. [6].