Optimizing Credit Risk Mitigation Effects of Collaterals Under Basel II

Marc Gürtler, Dirk Heithecker, and Martin Hibbeln

Department of Finance, TU Braunschweig, Germany
marc.guertler@tu-bs.de
d.heithecker@tu-bs.de
martin.hibbeln@tu-bs.de

Summary. One of the main differences in measuring capital requirements for credit risk due to the Internal Ratings Based Approach (IRBA) of the new regulatory Capital Standards (Basel II) is the enlarged acceptance of collaterals in order to reduce (supervisory) credit risk. Whereas in the Advanced IRBA own models for estimating the effects of collaterals can be used, in the Foundation IRBA concrete formulas are given. However, these regulations only explain the case of a single claim collateralized by a single asset or guarantee. If numerous collaterals are available for multiple loans, we receive an allocation problem of collaterals to loans with the objective of keeping regulatory capital low. In this article we model the corresponding optimization problem. Since numerical standard procedures often converge towards the wrong local minimum, we show how to apply evolutionary algorithms as well as simulated annealing.

1 Introduction

With starting of Basel II in 2007/2008 banks face significant changes in measuring credit risk when using the Foundation or Advanced IRBA in order to quantify the minimum (equity) capital needed to meet supervisory requirements [1]. Thus, regulatory credit risk of a loan is defined by the Unexpected Loss (UL) and the Expected Loss (EL), which both depend on the Probability of Default (PD) and the Loss Given Default (LGD). The variable PD quantifies the possibility of a potential default whereas the variable LGD measures the percentage loss in the event of default. The parameters PD and LGD not only depend on the debtor's individual ability to pay his liabilities but also on the debtor's collaterals and guarantees, respectively, to safeguard the bank against potential losses. Different from the Advanced IRBA, in the Foundation IRBA the supervisory board published specific rules how the risk mitigation effect on a single loan has to be measured.

2 Credit Risk Mitigation and Double Recovery

Concretely, under the Foundation IRBA two types of securities exist, collaterals and guarantees. The eligible collaterals are financial collaterals, receivables, real estate, and other physical assets. For each collateral assigned to a loan the bank has to determine its "risk adjusted" value that quantifies the fully collateralized part of the loan. The weighted sum of the reduced LGD of this fully secured part and of the LGD of the unsecured part defines the so called "effective" LGD of the loan. The risk mitigation effect of guarantees follows an approach of "partly" substituting both the risk parameters PD and LGD of the loan. Firstly, the capital requirements of the fully guaranteed part of the loan depend on the PD of the guarantor (substitution approach) or on the PD of both the guarantor and the debtor (double-default effect).[2] Secondly, the bank may use the LGD of the guarantor or the debtor. Thus, the effect of double recovery is not allowed.[1] Against this background, the collateral that is directly assigned to the loan can not be used to reduce the LGD of the guarantor.

3 Formulation of the Allocation Problem

In praxi multiple loans (of one or more debtors) are often secured by multiple collaterals and guarantees, i.e. we have J collaterals $j = \{1, \ldots, J\}$ and K guarantees $k = \{1, \ldots, K\}$ for I loans $i = \{1, \ldots, I\}$. Thus, we have to allocate these $J + K$ securities to the I loans. In order to keep the total capital requirements (CR) low, we aim to determine the optimal coefficients x_{ij} and x_{ik}, which assign the allocation of securities to loans in percent. To set up the optimization problem we need (default) risk weights $W_i^{(CR)}$ of a loan and $W_{ik}^{(CR)}$ of a guaranteed loan that are functions of the one-year probability of default PD_i of the debtor and PD_k of the guarantor. If we minimize CR, we have to account for (default) risk weights with respect to UL and EL. Further, we need the adjusted values C_{ij}^* and G_{ik}^* of the securities. We get[2]

$$W_i^{(CR)} = F_i \cdot SF + PD_i \ , W_{ik}^{(CR)} = \min\left(PD_k + F_{ik}^{Sub} \cdot SF; F_{ik}^{DD} \cdot SF\right) \quad (1)$$

$$C_{ij}^* = \max\left(0; \frac{C_j}{k_j^h} \cdot \left(1 - H_j^C - H_{ij}^{FX}\right) \cdot H_{ij}^{MM}\right) \text{ and} \quad (2)$$

$$G_{ik}^* = \max\left(0; G_k \cdot \left(1 - H_{ik}^{FX}\right) \cdot H_{ik}^{MM}\right). \quad (3)$$

We receive the minimization problem[3]

[1] See [1], paragraphs 284 (ii) and 303.
[2] For further explanation of the variables see Table 1.
[3] Again, see Table 1 for the variables. For simplification we set the current value of the exposure equal to EAD, that generally fits for typical bank loans. Further, we use $S(\cdot)$ as the indicator function with $S(s) = 1$ if statement s is true.