Periodic Timetable Optimization in Public Transport*

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Summary. „The timetable is the essence of the service offered by any provider of public transport.“ (Jonothan Tyler, CASPT 2006)

Despite this observation, in the practice of planning public transportation, only some months ago OR decision support has still been limited to operations planning (vehicle scheduling, duty scheduling, crew rostering). We describe the optimization techniques that were employed in computing the very first optimized timetable that went into daily service: the 2005 timetable of Berlin Underground. This timetables improved on both, the passenger travel times and the operating efficiency of the company.

The basic graph model, the Periodic Event Scheduling Problem (PESP), is known for 15 years and it had attracted many research groups. Nevertheless, we report on significant progress that has been made only recently on issues like solution strategies or modeling capabilities. The latter even includes the integration of further planning tasks in public transport, such as line planning.

On the theory side, we give a more precise notion of the asymptotical complexity of the PESP, by providing a MAXSNP-hardness proof as a kind of negative result. On the positive side, the design of more efficient algorithms gave rise to a much deeper understanding of cycle bases of graphs, another very hot topic in discrete mathematics during the last three years. In 2005, this culminated in both, drawing the complete map for the seven relevant classes of cycle bases, and the design of the fastest algorithms for the Minimum Directed Cycle Basis Problem and for the Minimum 2-Basis Problem.

The book version of this extended abstract is available as reference [8].

1 Timetabling

It is a very important competitive advantage of public transport to be much less expensive than a taxi service. This requires many passengers to share the same vehicle. Typically, this is achieved by offering public transport along fixed sets of routes, the lines. These serve as input to timetabling.

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There is a large toolbox of different types of timetables, which we introduce from the most general one to the most specialized one:

- timetables that are composed of individual trips,
- periodic timetables, i.e. the headway between any two successive trips of the same line is the same,
- symmetric periodic timetables, and
- so-called "Integrated Fixed-Interval Timetables."

Here, a periodic timetable is called *symmetric*, if for every passenger the transfer times that he faces during his outbound trip are identical to the transfer times during his return trip, which here is assumed to have the same route. In particular, the periodic timetables of most European national railway companies are indeed symmetric, because marketing departments consider this being a competitive advantage—at least in long-distance traffic.

Theorem 1 ([7]). *There exist example networks showing that* each *more specialized family of timetables causes a nominal loss in quantifiable criteria, such as average passenger waiting time.*

We are only aware of periodic timetables being able to clearly outweigh their nominal loss (when comparing with general irregular timetables) by adding benefit in qualitative criteria. Hence, in the remainder we focus on periodic timetables.

Typically, the period time T varies over the day. For instance, Berlin Underground distinguishes

- rush hour service ($T = 4$ minutes),
- „normal" service ($T = 5$ minutes),
- weak traffic service ($T = 10$ minutes, when retail shops are closed), and
- night service ($T = 15$ minutes, only on weekends).

Computing "the timetable" thus decomposes into computing a periodic timetable for each period time, and finally glue these together.

2 A Model for Periodic Timetabling

A literature review of different models for periodic scheduling reveals that the most promising earlier studies on medium-sized networks are based on the Periodic Event Scheduling Problem (PESP, [18]), see [17, 14, 9, 16]. The vertices in this graph model represent events, where an event $v \in V$ is either an arrival or a departure of a directed line in a specific station. A timetable π assigns to each vertex v a point in time $\pi_v \in [0,T)$ within the period time T. Constraints may then be given in the following form.

T-PERIODIC EVENT SCHEDULING PROBLEM (T-PESP)

Instance: A directed graph $D = (V,A)$ and vectors $\ell,u \in \mathbb{Q}^A$.
Task: Find a vector $\pi \in [0,T)^V$ that fulfills

$$(\pi_v - \pi_u - \ell_a) \bmod T \leq u_a - \ell_a \qquad (1)$$

(or $\pi_v - \pi_u \in [\ell_a,u_a]_T$, for short) for every arc $a = (u,v) \in A$, or decide that none exists.