ILP Models for a Nurse Scheduling Problem

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1 Introduction

The widely deplored shortage of qualified personal in the nursing sector all over Europe can be traced to a number of reasons among which the relatively short duration that personal remains on the job ranks prominently. Besides the high psychic pressure involved with many nursing jobs, the degree of job satisfaction is also decreased by irregular working hours and inflexible working schedules.

Many different types of nurse scheduling problems have been treated in the literature (see the excellent literature review by Burke et al. [1]). What basically all these versions have in common is that working shifts have to be assigned to nurses for a certain planning horizon (usually one month), such that the given demand and a large number of legal, departmental and organizational constraints are fulfilled.

The special type of nurse scheduling problem we deal with in this contribution arises in the university hospital of Graz (LKH Klinikum Graz), an institution with a nursing staff of more than 3000. Schedules are currently set up manually by the head nurses of each ward where a ward comprises 10 to 50 nurses. Our main objective is not minimiza tion of costs, but the generation of schedules which are perceived as fair and satisfactorily with regard to general and individual preferences.

The special features making this problem worth to be studied are the following. First of all, the list of available working shifts does not consist only of the traditional early, late and night shift, but of a large variety of overlapping shifts with different starting times and durations. Secondly, the daily demand profile splits each day in a number of demand periods, each with a given minimum number of nurses. The demand periods do not correspond, however, to the time partition caused by the available shifts. Thirdly, there is a significant proportion of part-timers including not only half-time employees, but also other fractions between 1/2 and 1. An interesting structural property is the preference for *blocks of duties*, which means that blocks of three or four working days followed by several days off are strongly preferred against single working days alternating with single days off. Additional difficulties result from the fact that individual preferences of the nurses should be taken into account and from the fact that in some planning periods no feasible solution might exist due to too many nurses on holidays or on sick leaves.

In this extended abstract we will describe the basic features of two mathematical models for the version of the nurse scheduling problem that showed up in our application. Further details will be presented in the full paper version.

2 Problem Description

A full description of all constraints that have to be taken into consideration when designing a feasible monthly schedule in our application is beyond the scope of this paper. Hence, we concentrate on the most important structural properties a schedule has to fulfill and simplify or even ignore some of the less important ones.

In our case, a set of n nurses must be scheduled for a 30 day planning horizon. The nurses are partitioned into four sets corresponding to the type of their contract, namely 1/2, 2/3, 3/4 and full employees.

Working time is assigned by shifts. For each available shift type the time period it covers is given where some shifts contain a break in between. Moreover, for each shift type it is known for which group of nurses it is available. E.g. a shift with 8 hours duration may only be assigned to a full-time or a 3/4-employed nurse, whereas a four hour shift may not be assigned to a full-time nurse. Night shifts always last for 12 hours and can be assigned to any type of nurse.

Demand is given by a partitioning of the day into periods in each of which a certain number of nurses is required to work. Unfortunately, the start and end time of shifts do not coincide with the endpoints of the demand periods which makes overstaffing unavoidable. A real-world example of shift types and demand data is given in Table 1.

Table 1. Shift and demand data of a hospital ward.

shift	start	end	break	hours
F24	06:30	14:30	-	8
F20	06:30	12:30	-	6
H18	09:00	18:00	12:00-13:00	8
H33	06:30	18:00	10:30-14:00	8
S14	11:00	19:00	-	8
S38	13:00	19:00	-	6
N8	18:45	06:45	-	12

demand	number of nurses	
period	weekdays	weekends
06:30-08:00	4	3
08:00-11:30	5	4
11:30-12:00	7	6
12:00-14:30	6	5
14:30-18:45	2	2
18:45-06:30	2	1

The following constraints are *hard constraints* that have to be fulfilled by any feasible schedule.

1. Every third weekend must be free.
2. The total working time may deviate from the required working time RWT (e.g., number of working days per month times eight hours for a full-time nurse) by at most 20 hours.
3. The working time per week is at most 60 hours.
4. Between two shifts of work there must be at least 11 hours of free time.
5. After working a sequence of night shifts, at least 2 free days must follow.